"Learning" Mathematics

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(a) Introduction

During a yeshiva high school education, one of the few disciplines where *lemudei kodesh* and *lemudei chol* studies naturally overlap is mathematics. Whereas the government decides what mathematics is essential basic knowledge for a productive citizen, halacha requires the mastery of some of these same fundamental skills for proper application and execution of mitzvos. In this vein, *Sefer HaChinuch,*[1] *Mitzva* 258, on the commandment of keeping honest measurements, comments that although the relevant Biblical verses discuss liquid and solid measures as well as fair scales, the commandment applies to all types of transactions that involve quantitative measurement. *Chinuch* concludes by offering a lengthy discussion of how this mitzva extends to the knowledge of geometry, including the following

> ויש לעיין בו גם כן בין העיגולין והריבועין ואלכסונין, ועוד הרבה עניינים המתבארים בספרי חכמת החשבון והגמטריאות בין זוית נצבה לזוית גרוחת וזוית חדה ואילו שלש צורותיהן כסדר ובין משולש שוה הצלעות והמשולש אשר שתי צלעותיו בלבד שוות והוא נקרא משולש שוה השוקים ובין משולש שאין מכל צלעותיות שוות והוא הנקרא מתחלף הצלעות ובין מרובע רבוע שוה למרובע ארוך...

We venture to say that a vast majority of today's serious *bnei Torah* not only do not know the mathematical formulae of these

[1] ספר החינוך was published in 13th-century Spain.

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figures, but probably cannot even translate the *Chinuch's* terms and expressions.[2]

The need for good mathematical skills, however, is not limited to halacha. The application of these *limudei chol* skills to many Gemaras is necessary for a proper appreciation of the Gemara. Despite these compelling reasons for bringing mathematical skills into the world of Torah, the tendency in yeshiva high schools today is to ignore these benefits, which often leads to one of two undesirable results:

- Right-wing yeshivos play down the secular curriculum, including mathematics, and the Gemaras requiring mathematical skills are often not properly understood.
- Modern Orthodox schools make sure that the mathematical skills are mastered, but Gemaras that present a quantitative analysis that seems to be at odds with these concepts may be viewed in a less than favorable light.

This paper starts with several elementary mathematical areas of study: number theory, geometry and algebra, and demonstrates in detail how they impact our understanding of a *sugya* in מסכת עירובין. We focus on how different *Rishonim* living at different times in societies with a spectrum of different mathematical skill levels offer their explanations of the text and at times express difficulty understanding the explanations of other *mefarshim*. For proper perspective, in footnotes, we also offer the interested reader comments on the historical development of mathematics across different societies and cultures. The reader will see that the acquisition of mathematical knowledge has not always followed a straight-line, upward trajectory through time, and that the *gedolim* of each era and geo-

2 "And one should also delve into the differences between circles, squares, diagonals and other things {shapes} that are explained in books of mathematics and geometry, between a right angle, an obtuse angle and an acute angle and between a triangle whose sides are all equal {English- equilateral triangle}, a triangle with only two equal sides which is called a triangle with equal legs {English- isosceles from Greek *isos* meaning equal and *skelos* meaning leg}, and a triangle all of whose sides are different which is called (a triangle) of uneven sides {English- scalene from Greek *skalenos* meaning uneven} and between a square and a rectangle."

graphic location typically were knowledgeable of the mathematics of their time and place, for better or for worse. They were also often the transmitters of mathematical knowledge to their contemporary Jewish as well as non-Jewish societies. These historical notes will help the reader understand why the particular commentators offered the explanations they did and will demonstrate that the functional mathematical sophistication of the *Chachmei HaTalmud* often exceeded that of some *Rishonim* and *Acharonim* who lived centuries later.

Whereas in the *sugya* in *Eruvin* all commentators regardless of their mathematical sophistication ultimately arrive at the same basic conclusions, this is not the case for a Mishna in כלאים that deals with an application of the same mathematical issues. We will cite the words of later commentators arguing factual quantitative differences with their predecessors based on mathematical realities. Although the later commentators express the utmost reverence for those who preceded them, they clearly express their unwillingness to accept statements they knew were mathematically wrong.[3] Along these lines we will argue that there is no need to spend time reviewing and explaining *Rishonic* applications of outdated and/or incorrect medieval mathematics. The example from *Kilayim* will also illustrate that mathematics is only a tool in understanding Gemara. The ultimate *pshat* must be based on one's sophistication in Torah study.

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3 Both *sugyas* we discuss are mathematically analyzed in W. M. Feldman, "Rabbinical Mathematics and Astronomy," Herman Press, 1931, albeit with a different objective. Feldman's paper includes an analysis of many other Gemaras and mathematical concepts and is an excellent reference work around which a Gemara-mathematics curriculum can be developed. Mathematical topics and *sugyas* discussed in Feldman include:

Eruvin 7b-8a, 76a	- Inscribed and Circumscribed Circles
Eruvin 43b	- Trigonometry
Eruvin 14b	- Solid Geometry

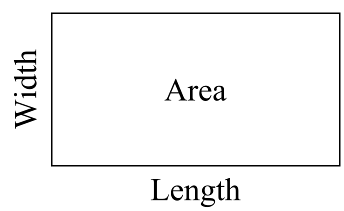
(b) Geometry, Number Theory and Algebra

(i) <u>Geometry</u>

Definition: Branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space. Geometry is one of the oldest mathematical sciences. Initially a body of practical knowledge concerning length, areas, and volumes, in the 3rd century BCE (early 2nd Temple era), geometry was put into an axiomatic form by Euclid.

<u>Example: Area of a Rectangle</u>

The Area of a rectangle = Length x Width (1)



Thus the Mishkan described in the Torah as having a courtyard of 50 by 100 Amos, encompasses an Area of 50 x 100 = 5,000 square Amos.

Rambam, פירש המשניות עירובין ב:ה, expresses this concept of Area and its calculation:

> וכבר ביארנו שתשבורת חצר המשכן שהיא בית סאתים חמשת אלפים אמה לפי שארכו מאה ורחבו חמשים וכל מקום שתשבורת שטחו חמשת אלפים אמה הרי הוא בית סאתים איך שתהיה תבניתו עגול או מרובע או משולש או שאר תבניות.

> We have previously explained that the area of a the courtyard of the Tabernacle is a *bais seasiam* of 5000 (square) Amos because its length is 100 and its width 50, and whenever the area of an expanse is 5000 (square) Amos it is a *bais seasaim* regardless of whether its shape is circular, square, triangular or any other shape.

Example: Dimensions of a Square בית סאתים

The Mishna[4] states that a square of a little more than 70 x 70 is equal in area to the Mishkan's courtyard which was a rectangular 50 x 100. Rashi, כג: ד"ה אלא, offers a detailed multi-step geometric analysis showing this equivalency. Today we would get the results by means of the square root function on a calculator, i.e. $\sqrt{5000} \approx 70.71$. Gandz[5] comments:

> Now this explanation of Rashi is no invention of his own. It is also found in the commentary of R. Hananel, and what is more important, the calculations given in the Palestinian Talmud quite evidently allude to such a procedure... There can be no doubt that the Babylonian Talmud... meant precisely that procedure which is so clearly expounded by Rashi.[6]

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4 עירובין **משנה** כג: ועוד אמר רבי יהודה בן בבא, הגינה והקרפף שהיא שבעים ושיירים על שבעים ושיירים, מוקפת גדר גבוה עשרה טפחים--מטלטלין בתוכה: ובלבד שיהא בה שומירה או בית דירה, או שתהא סמוכה לעיר... רבי עקיבה אומר, אפילו אין בה אחת מכל אלו, מטלטלין בתוכה--ובלבד שיהא בה שבעים ושיירים על שבעים ושיירים...

5 S. Gandz, "Studies in Hebrew Mathematics and Astronomy," *Proceedings of the American Academy for Jewish Research,* Vol. 9, (1938-1939), pp. 5–50.

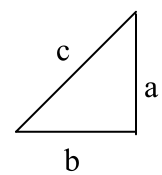
6 He characterizes this technique as "Oriental in its whole makeup" and shows how Al-Khuwarizmi used it to solve quadratic equations of the form $x^2 + 10x = 39$, and that it "has nothing to do with Euclidean geometry." Gandz says that while the main route of the transmission of science was:

Old Oriental (Babylonian-Egyptian) → Greek →Middle Oriental (Islam-Judaism) → Latin

"In certain branches of science... the Greek link was entirely eliminated. This was the case, for example, with algebra. I do not mean to say that the Greeks had no algebra:... The Europeans, however, did not learn their algebra from the Greeks, but from the Arabs; hence the Arabic name of algebra, and the Arabic terminology and method prevailing throughout the Middle Ages. The Arabs in their turn did not learn their algebra from Greek sources, but from those Syrian or Persian schools where algebra was cultivated as an old indigenous Babylonian science."

Example: Pythagorean[7] Theorem

For any right triangle, the length of the hypotenuse (in the picture below this length is c) squared equals the sum of the squares of the other two legs of the triangle (in the picture below the lengths of the vertical and horizontal legs are respectively a and b).



Thus: $c^2 = a^2 + b^2$ (2)

This ancient Greek formula is expressed by the *Tosfos Yom Tov, Eruvin* 2:5, in the following equivalent but more difficult formulation:

שכל אלכסון מרובע ארוך הוא גדר למרובע שרבועו כשני המרובעים שקו אורך ורוחב גדריהם.

The diagonal of any rectangle is the side of a square whose square value is equal to (the sum of) the two squares with sides of the length and the width {of the rectangle}.

(ii)Number Theory

Definition: A branch of pure mathematics devoted primarily to the study of the integers. Included in this topic is the study of rational and irrational numbers.

7 Pythagoras lived ca. 540 BCE. Gandz comments: "... the fact that the Pythagorean theorem is never mentioned or alluded to in the whole Talmudic literature, seems to prove lack of knowledge."

Topic	Examples
Integers	1, 15, 1002
Infinity	
Rational Numbers	
Fractions	1/2, 4/5, 1.527
Decimals: finite	.250, .367
Decimals: infinite repeating	.3333= 1/3, .9999 = 1, .232323= 23/99
Decimals: significant digits	$1/3 \approx .333$, $1/6 \approx .167$
	(Note: "$\approx$" means "is approximately")
Irrational Numbers	
Special Values	pi: $\pi \approx 3.14$
Square Roots	The square root of 2: $\sqrt{2} \approx 1.4142$
Decimals: infinite non-repeating	.23417243...

Example : Diagonal of a Square

עירובין נז .דאמר מר כל אמתא בריבוע אמתא ותרי חומשי באלכסונא.

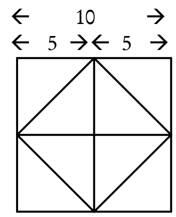
The master said every Amah in {the side of} a square has an Amah and two fifths in its diagonal

Based on the Pythagorean theorem, (2), a square of unit size has a hypotenuse of $\sqrt{2}$, i.e.:

$$c^2 = a^2 + b^2 = 1 + 1 = 2.$$

The Mishna's statement suggests that $c = \sqrt{2} = 1.4$. Tosfos offer an elegantly simple geometrical proof that this cannot be correct:

i) Draw a square of 10 x 10 with an area of 100. (See picture below.)
ii) Break it into four smaller 5 x 5 squares, each with an area of 25.
iii) Connect the midpoints of the 4 sides of the large square to form an inscribed square. This square is exactly half the size of the larger square in area (i.e., 50)
iv) The sides of the inscribed square are the diagonals of the 5 x 5 squares.
v) If 1.4 is exact, diagonals are 5*1.4 = 7 and the area of smaller square is 49 (not 50).



Tosfos conclude

> תוס׳ ד״ה כל. ואין החשבון מכוון ...אלא יש מעט יותר מאמתא ותרי חומשי באלכסונא.
>
> The calculation is inexact and there is a little more than 1.4 in the diagonal.

It is not clear from this language whether Tosfos mean

- √2 has an exact value but the Gemara is offering an approximation,[8] or
- √2 is irrational and an exact measurement is not possible.

Rambam, however, on this subject is unequivocal in that exactness in this case is impossible and that the number is irrational:

> And the thing about this count is, as I explained previously about the relationship between the circumference of a circle and its diameter,[9] because it too is impossible to ever be known exactly, only approximately. And this is not because of our shortcoming but because the nature of that number is such. And therefore it {the Mishna} said 70 Amos and some left over, because if you make the leftover five sevenths, the way I explained to you, and will multiply 70 and 5/7 by 70 and 5/7, it will result in a value of close to five thousand one and a half. (*Pirush Hamishnayos, Eruvin* 2:5)[10]

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8 Within this possibility we can also conjecture whether Tosfos mean the Gemara

- is suggesting that this approximation can be used for halachic purposes,
- wants the real value to be used but is writing it here to the nearest tenth.

9 I.e., π is not exactly 3. It is irrational.

10 אבל שטח מרובע בעל זויות ישרות שתשבורתו חמשת אלפים אמה אי אפשר לדעת צלע אותו השטח אלא בקירוב לפי שחמשת אלפים הוא מספר שאין בו שורש ושרשו בקירוב שבעים

In mathematical notation:

$$\sqrt{5000} = \sqrt{2500} * \sqrt{2} = 50\sqrt{2} \approx 50*1.4 + (\text{שירים})$$

Setting $\text{שירים} = 5/7$

$$(70 + 5/7)^2 = (70.7142857...)^2 = 5000.51020...$$

Rambam goes on to say that if we choose the שירים to be 2/3 (as he says the *Yerushalmi* does), then the area of the square will be 4,993 & 7/9:

$$(70 + 2/3)^2 = (70.66666...)^2 = 4{,}933.777....$$

Rambam, 1135–1204, born in Spain and later moved to Egypt, seemed to be well versed in the Greek and Arabic mathematics that is common knowledge today. Rambam makes multiple mathematical assertions without offering proofs as to their accuracy. We would assume that he thought anyone who read his commentary could verify its accuracy. *Rishonim* living centuries after Rambam, however, did not seem up to the mathematical challenge. For example, Bartenura,[11] *Eruvin* 2:5, comments

והרמבם בקש חשבונות רבים ולא ירדתי לסוף דעתו.

> And Rambam sought out many calculations and I do not understand his final intent.

Tosfos Yom Tov[12] explains that the Bartenura means not to disagree with Rambam but rather to encourage the reader to try to understand the Rambam. To fulfill Bartenura's request, Tosfos

אמה וחמש שביעית אמה והענין במספר הזה כמו שביארתי לך למעלה ביחס קוטר העיגול להקיפו לפי שגם הוא אי אפשר לעולם להגיע לידיעת שורש המספר "אלצם" שלו אלא בקירוב ואין זה מחמת חסרונינו אלא שטבע אותו מספר כך ולפיכך אמרו שבעים אמה ושירים לפי שאם תעשה את השירים האלו חמש שביעיות כמו שביארתי לך ותכפול שבעים אמה וחמש שביעיות בשבעים אמה וחמש שביעיות יהיו תוצאות המספר חמשת אלפים וחצי אחד בקרוב.

11 רב עובדיה בן אברהם מברטנורא born Bertinoro, Italy ca. 1445, died Yerushalayim ca. 1515.

12 Rav Yom-Tov Lipmann Heller, born Bavaria 1578, died Krakow 1654.

Yom Tov proceeds to offer three full pages of mathematical explanation of Rambam's one-quarter-page commentary on the Mishna.[13]

Example: Rambam above said 70 + 5/7 squared is close to 5,000 and one half. Using standard techniques, this calculation is done manually[14] as follows:

<u>Using fractions</u>:
Step 1: 70 + 5/7 = 495/7
Step 2: 495/7 * 495/7 = 495*495/49
Step 3:

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   495
 x 495
  2475
 4455
1980
245025
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Step 4[15]: 245,025/49 = 5,000.510204…

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13 Tosfos Yom Tov's explanation seems to support the following description of the state of the art of mathematics in 16th/17th Century Bavaria. "The calculation techniques in India had advanced substantially beyond the algorithms for multiplication and division, and decimal fractions that Europe was just beginning to get used to in the late 16th century CE. Though right from the time of Christoph Clavius, and the calendar reform of 1582, active efforts were being made to procure calendrical and mathematical knowledge from Indians, Arabs, and Chinese, Europeans had difficulty in understanding these texts." http://www.indianscience.org/essays/15-%20E--Navigation%20&%20Math.pdf

14 Almost everyone today would use a calculator to get the answer. In school, however, everyone is still taught to do it manually.

15 The final value is an infinite repeating decimal. For practical usage the solution would be written with 1 or 2 decimal places. The problem could also be done using decimals, i.e., 70+ 5/7 =70.714285… (a repeating decimal). The Step 3 multiplication requires that 70.714285… be rounded to as many significant digits as wanted and the process is complete. However, depending on where the decimal is rounded the solution can be as low as 4,998.49 (i.e. 70.7 x 70.7). Greater precision requires more decimal places and hence more manual multiplications.

Note: The final value is a decimal that infinitely repeats. For practical usage the solution would probably be given to 1 or 2 decimal places.

Tosfos Yom Tov on Steps 1 and 3: (Note: All calculations are in words. He never uses numerical figures.)

Step 1: Seventy and Five sevenths is Four Hundred and Ninety-Five sevenths.

Step 3:	i)	ה times ה	כה (5 units and 2 tens)	
	ii)	ה times צ	תן (4 hundreds and 5 tens)	
	iii)	ה times ת	two thousands	
	iv)	צ times ה	תן	
	v)	צ times צ	ח thousands & ק (8 thousands and 1 hundred)	
	vi)	צ times ת	ל״ו thousands (he explains why)	
	vii)	ת times ה	two thousands	
	viii)	ת times צ	ל״ו thousands	
	ix)	<u>ת times ת</u>	ק״ס thousands	(he explains why)
Summing the above[16]-			ה units	(from line i)
			י״ב tens	(from i, ii, iv)
			ט hundreds	(from ii, iv, v)
			רמד thousands	(from iii, v, vi, vii, viii, ix)

Since ten tens is one hundred and ten hundreds is one thousand, these numbers combine to רמה thousands and כ״ה, as above.

While we are not prepared to say that Tosfos Yom Tov did not know the shorter, more direct method presented first, the fact that he went into so much detail clearly indicates that he thought his reader did not know the more concise methodology.

[16] This is a slight variation of what Tosfos Yom Tov says in about a quarter page.

(iii) Algebra

Definition: The branch of mathematics concerning the study of the rules of operations and relations, and the constructions and concepts arising from them, including terms, polynomials, equations and algebraic structures. Elementary algebra introduces the concept of variables representing numbers.

Example: In the very same Mishna, Rambam explains that Rebbe Eliezer says that for a rectangle to meet the בית סאתים requirement, its diagonal must be no more than twice its width, and Rambam offers the dimensions of such a rectangle:

כגון שיהיה אורך השטח תשעים ושלשה ושליש תשיעית והרוחב חמשים ושלשה ושלש רבעים יהיה האלכסון מאה ושבעה וחצי.

i.e., a 93 1/27 x 53 3/4 has a diagonal of 107½.

Algebraically

We know that i) Length x Width = 5,000 → Area

ii) Diagonal = 2W → Diagonal is twice the Width

iii) $D^2 = L^2 + W^2$ → Pythagorean Relationship

Hence $(2W)^2 = (5000/W)^2 + W^2$ → Substitution from i) –iii)

$3W^2 = (5000/W)^2$ → Regroup

$\sqrt{3}W = 5000/W$ → Square Root of both sides

$W^2 \approx 5{,}000/\sqrt{3} = 2{,}886.75$

$W \approx 53.73$

$L \approx 93.06$

$D \approx 107.45$

Tosfos Yom Tov

He does not show how the dimensions of the rectangle are a priori determined but does attempt to show, using the method described in the previous page, that a rectangle with dimensions that Rambam gives has a diagonal that is twice its width. He concludes his long exposition with the comment that based on his calculations there is a slight flaw in Rambam's numbers

וכן הקרוב אלי כי טעות סופר הוא בלשון הרמבם וצריך להיו ק״ז ורביע.

i.e., the diagonal is 107 1/4 not 107½.

Tosfos Yom Tov's final statement is puzzling because we have shown algebraically that the answer is closer to 107.5 than to 107.25. Also, if Tosfos Yom Tov is correct, then the length of the diagonal is "considerably" less than twice the width. It turns out that the solution to the problem is indeed an incorrect text but not the one Tosfos Yom Tov thought. In many standard editions of *Pirush Hamishnayos*, the width of the rectangle is given as 53 1/3, not 53 3/4. Using this for the width, the diagonal indeed comes out to be 107.24 as Tosfos Yom Yov suggests. However, as we have proven algebraically, the correct width given by Rambam is 53 3/4, and this correction has already been noted by the Gra.[17] Had Tosfos Yom Tov solved the quadratic equation as we demonstrated, he would have realized the correct values as given by the Rambam.

It does, however, seem logical that Rambam would be familiar with the algebraic formulation. This technique was known in Arabic circles in Rambam's times. Interestingly, the first complete solution of the quadratic equation known in Europe of

$$x^2 - ax + b = 0$$

a more complex algebraic expression, appeared in *Ḥibbūr ha-meshīḥah we-ha-tishboret* ("Treatise on Measurement and Calculation"), a Hebrew treatise on Islamic algebra and practical geometry by Abraham bar Ḥiyya ha-Nasi in the early 12th Century.[18,19]

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17 ר' אליהו בן שלמה זלמן born 1720 died in Vilna 1797. Feldman comments, "Obviously there is some error in Maimonides' calculation, ...Possibly, the 53 1/3 of Maimonides is a misprint for 53 2/3." He does not mention Tosfos Yom Tov nor the Gra's correction.

18 Born in Barcelona 1070, died in Narbonne ca. 1140. Known as Savasorda, he was also an astronomer and philosopher and lived one generation before Rambam.

19 Concerning the relationship between Arabic and Jewish mathematical works, Gandz suggests several routes:

Greek → Hebrew → Syriac or Persian → Arabic, or
Greek → Syriac → Arabic → Hebrew

And adds: "Like the Persian and the Christian cultures, Hebrew culture stood at the beginning and at the end of the classic period of Islam. The Hebrews were the teachers and disciples of the Muslims. Teachers at the cradle of Islam, and disciples when it reached its maturity."

(c) Mathematical Knowledge is Necessary But Not Sufficient

Mishna כלאים ה:ה presents a set of rules that determines under what conditions vines and vegetables planted near each other constitute כלאים.

> ה:ה הנוטע ירק בכרם, והמקיים--הרי זה מקדש ארבעים וחמש גפנים ; אימתיי--בזמן שהן נטועות על ארבע ארבע, ועל חמש חמש. היו נטועות על שש שש ,ועל שבע שבע--הרי זה מקדש שש עשרה אמה לכל רוח, עגולות אבל לא מרובעות.
>
> If one plants vegetables or retains them in a vineyard, he renders forbidden forty-five vines. When? If they are planted four or five apart. If they were planted six or seven apart, he renders forbidden sixteen cubits in every direction in a circle, not in a square.

Without going into a detailed explanation of the Mishna,[20] suffice it to say that knowledge of basic mathematical concepts we have discussed before (particularly that of the length of a diagonal) is a prerequisite for understanding the Mishna. Rabbeinu Shimshon[21] and Rambam offer different explanations based on a different understanding of basic mathematical formulae. Tosfos Yom Tov, as he did in *Eruvin*, here too has a multi-page explanation of the Mishna. He begins his commentary by saying that he will start with a simple explanation of the Mishna and ultimately the reader will see that the Rash in this instance was completely (mathematically) wrong.[22] Similarly, Bartenura, after offering a long explanation of the Mishna along the lines of the Rambam, concludes that since the Rosh[23] and others have affirmed the correct-

20 See Feldman.

21 I.e., רבינו שמשון משאנץ commentator on Mishna. He was an early French Tosafist (1150–1230). Rosh (ראש) said of him that only Rabbeinu Tam and Rabbi Isaac ben Samuel exercised greater influence upon Talmudical studies in France and in Germany during the 13th century.

22 שבתחילה אפרש המשנה בלשון קצרה כדי שתהא מפורשת לכל במעט עיון ואח"כ אעתיק לשון ואח"כ אבאר אותם אחת לאחת בבאור מספיק מאוד וממילא רווחא ... הרמבם בפירושו שמעתתא שדברי הר"ש מנוחתו כבוד ובמקומו מונח הם בטלים במקום הזה לא שרירין ולא קיימין.

23 אשר בן יחיאל born 1250 in western Germany and died 1327 in Toledo Spain.

ness of Rambam's explanation, he will not bother giving Rash's explanation.[24] Bartenura's reference to the Rosh's affirmation of Rambam's explanation is interesting. כסף משנה, הלכות כלאים ו:א-ב, writes that Rosh himself, not knowing whose explanation of the Mishna was correct, turned to R' Yisrael, a scientifically knowledgeable person, for help and R' Yisrael told him that Rambam's explanation is (mathematically) correct and that the Rosh should not deviate from it.[25]

Thus, Rosh's decision to side with Rambam was based not on his own analysis but on his reliance on a capable mathematician. Tosfos Yom Tov seems to have been the first traditional commentator willing and able to offer a detailed refutation of Rabbeinu Shimshon's approach. Tosfos Yom Tov, interestingly, does not limit his detailed refutation to the facts. He offers a blistering attack on Rabbeinu Shimshon's methodology:

> ולמה לא עשה לו הר"ש צורה מוחשת לראות ולהבין בה כאשר לא האמין לדברי חכמי המידות?
>
> Why didn't the Rash make drawings to see and understand {the mathematics} when he did not believe the words of measurement experts?

Tosfos Yom Tov here is referring to Rabbeinu Shimshon's acceptance of the Pythagorean Theorem for a square but not for a rectangle based on his explanation of the Mishna.[26]

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24 ולפי שראיתי אחד מהחכמים הקדמונים שכתב על זה הפירש הן מהדברים שנאמרו למשה וג"כ הרא"ש ז"ל הסכים לזה הפירוש ודחה פי' ה"ר שמשון ז"ל לא הוזקקתי לכתוב פירוש ה"ר שמשון ז"ל על משנה זו.

25 ומצאתי כתוב ששאל הרא"ש את ה"ר ישראל שהיה חכם בחכמות על דברי מי יש לסמוך והשיב לו כלשון הזה אדוני ומורי תרב גדולתך כגודל כשרון פעולתך עיינתי בפירוש משנת הנוטע ירק בכרם להר"מ ז"ל וראיתי כי הפליא עצה הגדיל תושיה בפירוש אבל נראה לפי עניות דעתי כי האריך בה הרבה והלשון אינו מחוור כפי הצורךכי נראה סתירה בדבריו לפי שאינו מעיין בהם עיון דק אבל כולם נכוחים למבין והענין אין לנטות ממנו ימין ושמאל.

26 Rash writes:
ואע"פ שהכחתי הדבר לרבוע שארכו כרחבו אין לנו הוכחה לרבוע שארכו יותר על רחבו ועל כרחן ליתא להאי כללא דהא אלכסון של ל' על כ' אין עולה כי אם ל"ב כדמוכח מתניתן כדפרישית...
Based on Pythagorean Theorem the diagonal is $\sqrt{30^2 + 20^2} = \sqrt{1300} = 36.06$, not 32 as Rash claimed.

Tosfos Yom Tov responds that had Rabbeinu Shimshon drawn such a rectangle he would have seen that the measurement experts were correct. Tosfos Yom Tov concludes:

> ואם הוקשה לו משנתינו לא מפני כן יכחיש מה שנראה לעינים[27] ואפילו דברי חכמי המידות לא הוה ליה להכחיש כי כל דבריהם בנוים על מופתים חזקים אשר א״א לסותרם בשום פנים...
>
> And if he {Rash} had a problem understanding the Mishna, that is no reason to reject what is supported by sight. And he should not have rejected the words of the measurement experts, because everything they say is based on strong evidence that cannot be refuted.

Having offered his analysis of the explanations given by previous generations of commentators, Tosfos Yom Tov asserts that his confirmation of Rambam's mathematics is sufficient only to reject Rav Shimshon's explanation of the Mishna. It does not mean that Rambam's explanation of the Mishna's intentions is correct. Tosfos Yom Tov based on other grounds disagrees with Rambam's ultimate explanation of the Mishna and concludes with a new explanation of his own.[28]

Thus, we have to deal here with a Mishna that cryptically states a rule, and presume that all existing factually correct explanations have linguistic or textual problems. This is quite usual in the world of Torah and requires a Torah scholar to resolve and/or offer a halachically binding ruling. In this particular case, because of problems he perceived in Rambam's explanation, Tosfos Yom Tov offers his own new explanation of the Mishna. However, מרכבת המשנה refutes all of Tosfos Yom Tov's objections to Rambam's explanation and feels that it is correct. Clearly, to offer a definitive explanation of the Mishna requires someone well versed in Torah as well as

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27 Tosfos, עירובין עו: ד״ה ורבי, use a similar expression to reject an interpretation of a statement made by דייני דקיסרי: וקשה היך טעו דייני דקיסרי הא קא חזינן דלאו הכי הוא.

28 ואנחנו לא נדע מה נעשה ואל מי מן מקודשים נפנה בפירש משנתינו אחרי שדברי כל המפרשים זכר כולם לחיי העולם הבא אין באחד מהם מה שמספיק שתנוח דעת כל משכיל עליו כי דברי הר״ש דחויין מעיקרן ודברי הרמבם גם המה לא יתכנו בפירש המשנה כאשר הוכחנו וכאשר נראה מדברי חיבורו שהוצרך לשנות הלשון והמליצה מפני הקושי אשר הרגיש בעצמו מה שיפול במתשנתינו לפי דרכו...

in mathematics. Rosh did not have the mathematical knowledge and felt he could not offer a viable opinion. However, knowing mathematics is a necessary but not sufficient condition for understanding the Gemara. It is questionable whether the mathematician the Rosh consulted, while able to understand the quantitative component, was enough of a Torah scholar to offer a definitive opinion on the Mishna's intent.

Suggestions on How and When Mathematics Is to be Taught

How and when the necessary mathematics skills are taught should depend on the school. Schools offering formal high school mathematics courses in their secular studies program can present the *sugyos* we have touched upon above[29] in a semester-long hour period after the underlying mathematics have been mastered in the *limudei chol* studies. Yeshivos not offering high school–level mathematics courses will either have to rely on rebbeim who are sufficiently mathematically literate to incorporate this material into their *shiurim*, or perhaps utilize visiting lecturers who can assist with selected *shiurim*. There are many available שאלות ותשובות ספרים, מפרשי הש"ס and journal articles that discuss a wide variety of Talmudic sources from a mathematical perspective.[30] It should not be difficult to put together high school–level Gemara/mathematics classes that are enlightening, stimulating, challenging and enjoyable. The main thing to ensure is that the mathematics material be taught sooner rather than later in a student's education. Studies show that mathematics, like languages, is most easily learned when someone is young, and delaying the introduction of these skills until a child is older is a recipe for failure.[31] ☙

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29 As well as other mathematical areas of study, e.g., statistics, probability, limits, modular arithmetic …

30 E.g., רש"ש ב"ב יד: ד"ה גמרא, ש"ות חות יאיר סימן קעב'.

31 E.g., see <http://www.youtube.com/watch?v=jNwAv6zpmh4> 18:00–19:10. To demonstrate the need for early mathematics education, the speaker cites a report from a conference at the Technion where the principal of the Jerusalem College of Technology reported that more than 50% of male charedim accepted in the college prep programs in technical fields drop out before graduation, while only 5% to 7% of the women do.