A Statistical Analysis of the Conjunction of Tishrei: The Sin of ADU

By: **SHELDON EPSTEIN, YONAH WILAMOWSKY,
     BERNARD DICKMAN and MAYER WEISS**

Introduction

The start of the month of *Tishrei* (Rosh Hashanah, R"H) in the Jewish Lunar Calendar is related to the time of conjunction[1] of the new moon. In reality, the time between successive conjunctions fluctuates, and averages 29 days, 12 hours, and 793 *Chalakim* (i.e., 44 minutes and 3⅓ seconds).[2] To simplify calendrical calculations, the *Molad* rather than the actual conjunction is used. The *Molad* assumes that the time between successive conjunctions is a constant 29D:12H:793C.[3]

Perhaps the best-known acronym concerning the designation of R"H is לא אדו ראש (*Lo ADU Rosh*), i.e., the 1st of *Tishrei* can never be Sunday, Wednesday or Friday.[4] The almost equally well known reason for this is:

If the 1st day of *Tishrei* were:

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[1] When the earth, moon and sun are approximately in a straight line.

[2] For a discussion of this value, see Epstein, Dickman and Wilamowsky, "A 5765 Anomaly," *Tradition*, Vol. 38, No. 3, pp. 40–59, Fall 2004.

[3] רמבם הלכות ק"הח ו:ג משיתקבץ הירח והחמה לפי חשבון זה עד שיתקבצו פעם שנייה במהלכם האמצעי תשעה ועשרים יום ושתים עשרה שעות שלשים מיום מתחילת לילו, ושבע מאות שלשה ותשעים חלקים משעת שלוש עשרה וזה הוא הזמן שבין כל מולד ומולד וזה הוא חודשה של לבנה.

[4] א for the 1st day of the week, ד for the 4th day of the week, and ו for the 6th.

Sheldon Epstein is a Professor of Quantitative Analysis at Seton Hall University and teaches in the Seton Hall - Yeshivah Gedolah of Marine Park Brooklyn (Rabbi Avraham Zucker, *Rosh HaYeshivah*) ITV Graduate Program.

Yonah Wilamowsky is a Professor of Quantitative Analysis at Seton Hall University.

Bernard Dickman is a Professor of Quantitative Analysis at Hofstra University.

Mayer Weiss is an MBA student at Seton Hall University's Graduate School of Business, located in South Orange, New Jersey.

- Sunday, then Hoshana Rabbah would be on Shabbos and there would be no *aravah* ceremony.
- Wednesday or Friday, then Yom Kippur would be on Friday or Sunday. This would result in consecutive days with severe restrictions on burials and food preparation (מתיא and ירקא).[5]

Rambam offers an entirely different reason for excluding ADU days.[6] He asserts that by allowing one day of the week to be R"H and then alternating successive days of the week in which R"H cannot/can occur, discrepancies between the actual conjunction and the *Molad* are reconciled. Thus, starting with Tuesday as a day on which R"H can occur and alternating successive days of the week in the manner described results in R"H never being on ADU. Rambam, however, does not explain:

- How alternating allowable and not allowable sequential days makes it more likely for the resulting calendar to be more consistent with the actual conjunction,
- Why the alternating routine is started on Tuesday, thus resulting in the elimination of ADU rather than starting it on another day and ending with a totally different combination of allowable and non-allowable days.[7]

Ravad, in a caustic gloss, rejects Rambam's explanation and asks "What sin did ADU commit" that the true conjunction never occurs in

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5   These reasons are alluded to in *Rosh Hashanah* 20a and *Succah* 43a but not with respect to our fixed calendar system. Ravad enunciates it clearly in *K"H* 7: 7. משום יום ערבה שלא יבוא בשבת ומשום יום הכיפורים שלא יבוא לא בערב שבת ולא במוצאי שבת.

6   ומפני מה אין קובעין בחשבון זה בימי אד"ו--לפי שהחשבון הזה הוא לקיבוץ הירח והשמש בהילוכם האמצעי, לא במקומם האמיתי, כמו שהודענו לפיכך עשו יום קביעה ויום דחייה, כדי לפגוע ביום הקיבוץ האמיתי. כיצד--בשלישי קובעין, ברביעי דוחין, בחמישי קובעין, בשישי דוחין, בשבת קובעין, באחד בשבת דוחין, בשני קובעין (ק"ה ז:ז).
    Why is it not established when it falls on ADU? Because these calculations determine the average conjunction of the sun and the moon, not their actual true position, as explained. Therefore, they instituted that one day it could be established and on the following day it would be postponed, in order to achieve the day of the true conjunction. How? Tuesday, we establish; Wednesday, we postpone it; Thursday, we establish; Friday, we postpone; Shabbos we establish; Sunday, we postpone; Monday, we establish.

7   E.g., had we started the alternating system with Sunday being a day on which R"H can occur, the result would be R"H occurring on Sunday, Tuesday, Thursday or Shabbos, but never on Monday, Wednesday or Friday (i.e., BDU).

these days?[8] Dr. Hugo Mandlebaum[9] quotes the *Shulchan Aruch Ha-Asid* (Rabbi Yechiel Michel Epstein) explanation of Rambam as follows:

> As Hashem created the moon, He created it in such a fashion, that one day it travels with true speed, and the day after with a mean speed, and the day after that again with true speed, and so on, day after day...

Mandlebaum's response to this is:

> With all due respect to Rabbi Epstein, one should not invent a *metzi'us* (situation) to fit a desired answer.

In this paper we demonstrate that with respect to the actual lunar conjunctions, the days of ADU can be shown to differ quantitatively from the other days of the week. While we do not offer a reason for the difference, we discuss how this difference may help us better understand Rambam's assertion that the ADU deferrals reconcile the conjunction/ *Molad* differences.

Variability of the True Conjunction

For any given month, the actual time between two successive *true* conjunctions in the 20th and 21st centuries ranges between approximately 29 days 6.5 hours, and 29 days 20 hours.[10] A detailed analysis of the trend of the relationship between the *Molad* (calendric conjunction) and true conjunctions is given by Mandlebaum. Based on data from 1943 through 1974, he demonstrates that the differences between these two conjunctions, (i.e., *Molad* — true conjunction), form sinusoidal curve cycles that:

- Vary in length from 13 to 16 months;

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8   א״א מפני שהמחבר הזה מתפאר בחכמה הזאת והוא בעיניו שהגיע לתכליתה ואני איני מאנשיה
    כי גם רבותי לא הגיעו אליה ע״כ לא נכנסתי בדבריו לבדוק אחריו אך כשפגעתי בדבר הזה
    שכתב נפלא בעיני הפלא ופלא ואם יהיה המולד בבגה״ז אל הדרך האמצעי למה לא ידחה
    למחרתו אל המולד האמיתי ולדבריו אין ראוי לעולם לקובעו ביום מולדו ומה חטא אד״ו שלא
   .יהיה בו המולד לעולם באמיתי ולעולם ידחה ומה זכה בגה״ז שיהיה בו ולא ידחה

9   Mandlebaum, "The Problem of Molad Tohu", *Proceedings of the Association of Orthodox Jewish Scientists*, 1976, Vol. 3–4, pp. 65–84.

10 MEEUS, *Astronomical Algorithms,* 1991, p. 324. Loewinger, *Parshat Bo, Daf Shvui,* No. 116. For the period 1000 BCE until 4000 CE, it ranged from a low of 29 days, 6 hours and 26 minutes (in 302 BCE) to a high of 29 days, 20 hours and 6 minutes (in 400 BCE). (See <http://snahle.tripod.com/ moon.htm>.)

- Have positive and negative segments;[11]
- Vary significantly in terms of maximum and minimum values; and
- Are not symmetrical, i.e., the absolute value of a cycle maximum can vary significantly from the absolute value of its cycle minimum point.

For the 32 years he analyzed, Mandlebaum offers the 1946 cycle and 1969 cycle (Figure 1) as the two most extreme cycles in terms of the magnitude of the difference between the maximum and minimum points, i.e., 15.7 vs –10.8, and 7.7 vs. –3.9.

Figure 1

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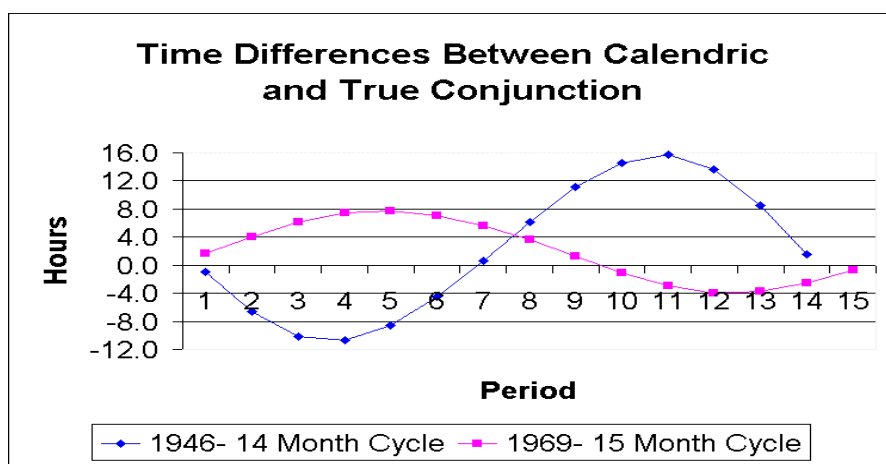
[11] A cycle is a sequence of difference points that begin with the first in a series of positive (or negative) differences and continues through the negative (or positive) differences until it returns once again to a positive (or negative) difference.

Are All Days Created Equal?

Because the *Molad* of a month is based on a calculation that repeatedly adds the same number, it seems intuitively reasonable that it is equally likely that the *Molad* of *Tishrei* would occur in any day of the week (i.e., Uniformly Distributed). However, because of the fluctuation of the inter-conjunction time from month to month, it is by no means obvious how the actual conjunction times are distributed between the days of the week. To test our assumption of the Uniform Distribution of the *Molad* over the days of the week and to see if any pattern is evident in the distribution of the actual conjunction over the days of the week, we initially reviewed the most recent 70 years of data[12] for each set of values. The results are given in Table 1 and pictorially presented in Figure 2.

Table 1
Distribution of the Conjunction[13] and *Molad* 1946–2015

	Conjunction	Molad
Sunday	9	9
Monday	11	10
Tuesday	10	11
Wednesday	12	10
Thursday	7	10
Friday	12	10
Shabbos	9	10
Total	70	70

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[12]   The source for the true conjunction time is: US Naval Observatory Website: <http://aa.usno.navy.mil/data/docs/MoonPhase.php>. The data from this site had to be adjusted from: i) Greenwich Mean Time to Jerusalem Standard Time; ii) standard convention of new day starting at midnight to Jewish convention of new day starting at sunset. Note that at *Tishrei* time the sun sets very close to 6 pm.

[13]   See Appendix 1 for a full listing of the data.

Figure 2
Distribution of the Conjunction and *Molad* 1946–2015
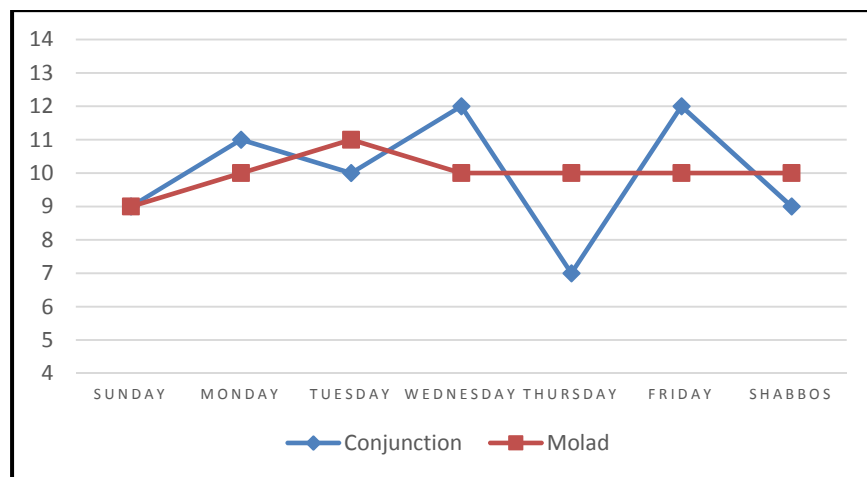
Our assumption about the distribution of the *Molad* over the days of the week is consistent with the data as 5 of the 7 days appear the expected 10 times and the other two days are one more or less than expected (i.e., 9 and 11 respectively). The distribution of the actual conjunction shows considerable variability ranging from a low of 7 for Thursday to a high of 12 for Wednesday and Friday. Although nothing stood out about Sunday (the A of ADU), the DU days of ADU (i.e., Wednesday and Friday) appear disproportionately overrepresented (i.e., 20% more than expected) as they represent only 2/7 (28.6%) of the days of the week but appear 24 times (34.3%) out of the total 70 observations. To see whether this frequency distribution was a localized current phenomenon, we reviewed the conjunction and *Molad* data for a 400 year period,[14] 1700 through 2099. The results are given in the next section.

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[14]   The years to be included in the study were chosen because they are the years for which the True Conjunction data is available on the Naval Observatory website.

Analysis of 400 Years of Conjunction and *Molad* Data[15]

Table 2 summarizes the results of a 400-year review of conjunction and *Molad* data.

Table 2
Distribution of the Conjunction and *Molad* 1700–2099

	Conjunction	*Molad*
Sunday	67	59
Monday	57	55
Tuesday	52	58
Wednesday	67	57
Thursday	44	57
Friday	69	58
Shabbos	44	56
Total	400	400

Table 2 confirms and extends the implications of Table l. The results are dramatically highlighted in Figure 3. With respect to the *Molad,* note the almost flat horizontal line of the graph. Even more strikingly, note how the days of ADU: represent the 3 peak points on the graph; are almost exactly the same; and the graph significantly declines after each of them. These 3 days represent 42.9% of the days of the week but 50.8% (203 out of 400) of the data points (18% more than expected). In general, the distribution of the conjunction among the days of the week appears to be broken into three groups:

Sunday, Wednesday, Friday — High Frequency
Monday, Tuesday — Intermediate Frequency
Thursday, Shabbos — Low Frequency.

For a statistical analysis of these results, see Appendix 2.

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[15] The raw data is available on line at <www.Hakirah.org/vol19_400Years.xls>. We note that based on Mandlebaum's analysis of the relationship between conjunction and *Molad,* it is possible that the *Molad* occurs the day before the conjunction. However, this occurred only 6 times in the 400 year period, i.e., 1749, 1765, 1872, 1934, 1996 and 2094. While there were many cases of the *Molad* coming before the conjunction, in the overwhelming number of cases they both were on the same day.

Figure 3
Distribution of the Conjunction and *Molad* 1700–2099
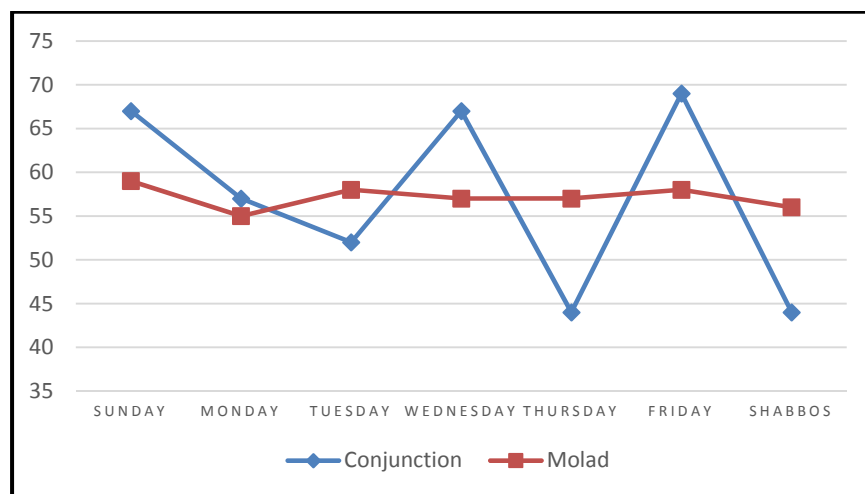
Discussion and Implications

As we said in the Introductory Section, Rambam highlighted the ADU days as being different from the other days but offered no explanation of how or why. Our presentation demonstrates that in fact the *metzi'us* (see Mandlebaum, above) is that Sunday, Wednesday and Friday *are* different than the other days in the week vis-à-vis the actual conjunction. Furthermore, whereas Ravad mocks Rambam for allegedly saying that the true conjunction seldom appears in these days, we have shown that to the contrary it is "more" likely that the conjunction occurs in ADU. The questions that remain are how and why all of these observations justify avoiding declaring R"H on these days. In this final section we will offer some preliminary thoughts.[16]

It is informative in studying our current fixed calendar system to briefly review the "sighting" system (ראיה) which preceded it. Rambam (*K"H* 5:1–3) says that the former system is the preferred one and it was in place from the time of the Exodus until the days of Abaya and Rava when

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16   Ultimately whatever we suggest also has to address the other major calendar rule: If the *Molad* occurs on Monday, Tuesday, Thursday or Shabbos
   - Before noon: R"H is that day
   - After noon: R"H is pushed off to the first day that is not ADU.
   We have not yet analyzed the conjunction frequency across the hours of the day.

the Sanhedrin ceased to exist. In the sighting system, the new month begins at the time of the first physical observation of the new moon, with moonrise and moonset occurring in very close proximity to sunset. First sighting is one of several alternate ways of determining the start of a new month in a lunar system. Some societies in the ancient world used last sighting (i.e., the last time the old moon was confirmed, seen) while others used the time of conjunction. Rambam explains, that even when "first sighting" was the system in place, the most sophisticated calculations for the time of the true conjunction were conducted to make sure that the sightings that were reported were possible (Rambam, *K"H* 1:6).[17] Since first visibility never takes place on the day of the conjunction, the day of conjunction could not be Rosh Chodesh or R"H. Thus, even though they knew the exact time of the conjunction they preferred to start the new month up to a day or two later[18] — when the first sighting took place.[19]

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[17] Rabbeinu Bachaya, *Shemos* 12:1 disagrees with Rambam. He cites Rabbeinu Chananel who maintains that calculation was the intended approach and "sighting" began in Tannaic times.

וענין החודש הזה לכם ראש חדשים אין כונת הכתוב להזהיר אותנו לעבר שנים ולקבוע חדשים על פי ראיית הלבנה כי בקביעות החדשים אין עיקר בתורה לחוש לראית הלבנה אם תראה מוטב ואם לא תראה ביום הקביעות אלא קודם לכן או אחרי כן ליום או יומים אין אנו חוששים כי לא נצטוינו בתורה לקבוע החדשים על פי ראית הלבנה כי אם על פי חשבון וכתב רבינו חננאל ז"ל קביעות החדשים אינו אלא על פי החשבון לא על פי ראית הלבנה והראיה שכל ארבעים שנה שהיו ישראל במדבר היה הענן מכסה אותם ביום ועמוד האש לילה ולא ראו בכלם שמש ביום ולא ירח בלילה והוא שאמר הכתוב (נחמיה ט) ואתה ברחמיך הרבים לא עזבתם במדבר את עמוד הענן לא סר מעליהם יומם להנחותם בהדרך ואת עמוד האש לילה להאיר להם ומהיכן היו קובעים חדשים על פי ראית הלבנה אלא בודאי עקר המצוה בכתוב על פיה חשבון.

[18] Rambam K"H 1:3. Loewinger, *Parshat Bo, Daf Shvui*, no. 116, section 2, writes:

"The first possible sighting, even under exceptionally good conditions, can occur only after about 13 hours (with optical aid) or after 15 hours (unaided vision) from the time of the *true Molad.*"

Thus, "sighting" never takes place on the conjunction day but could occur at sunset at the end of the conjunction day if the conjunction was before 3:00 am (i.e., 15 hours before sunset at 6:00 pm). If the conjunction is after 3:00 am the first sighting occurs at sunset of the day after the conjunction. We have not as yet reviewed the probability of it occurring before or after 3:00 am. See footnote 16.

[19] According to Rabbeinu Bachaya's approach it is possible that the beginning of the new month should be determined solely by time of conjunction but since a month cannot begin in the middle of a day, the start of the month is delayed to the day after conjunction so that the entire day, from a lunar perspective, is in this new month.

Appendix 1
Conjunction and *Molad* of Tishrei 5706–5775 (1946–2015)

Year	Conjunction Day	*Molad* Day	Yea	Conjunction Day	*Molad* Day
5706	Thursday	Friday	5741	Tuesday	Wednesday
5707	Wednesday	Thursday	5742	Monday	Tuesday
5708	Monday	Monday	5743	Friday	Shabbos
5709	Sunday	Sunday	5744	Wednesday	Wednesday
5710	Thursday	Thursday	5745	Tuesday	Tuesday
5711	Tuesday	Tuesday	5746	Sunday	Sunday
5712	Monday	Monday	5747	Shabbos	Shabbos
5713	Friday	Friday	5748	Wednesday	Wednesday
5714	Tuesday	Tuesday	5749	Sunday	Sunday
5715	Monday	Monday	5750	Shabbos	Shabbos
5716	Friday	Shabbos	5751	Wednesday	Thursday
5717	Wednesday	Wednesday	5752	Sunday	Monday
5718	Tuesday	Tuesday	5753	Shabbos	Sunday
5719	Shabbos	Shabbos	5754	Thursday	Thursday
5720	Friday	Friday	5755	Tuesday	Tuesday
5721	Wednesday	Wednesday	5756	Monday	Sunday
5722	Sunday	Sunday	5757	Friday	Friday
5723	Shabbos	Shabbos	5758	Thursday	Thursday
5724	Wednesday	Wednesday	5759	Monday	Monday
5725	Sunday	Monday	5760	Friday	Friday
5726	Shabbos	Sunday	5761	Thursday	Thursday
5727	Thursday	Thursday	5762	Monday	Tuesday
5728	Wednesday	Wednesday	5763	Shabbos	Shabbos
5729	Sunday	Sunday	5764	Friday	Friday
5730	Friday	Friday	5765	Tuesday	Tuesday
5731	Wednesday	Wednesday	5766	Monday	Monday
5732	Sunday	Monday	5767	Friday	Shabbos
5733	Friday	Friday	5768	Tuesday	Wednesday
5734	Wednesday	Thursday	5769	Monday	Tuesday
5735	Monday	Monday	5770	Shabbos	Shabbos
5736	Shabbos	Shabbos	5771	Wednesday	Thursday
5737	Friday	Friday	5772	Tuesday	Tuesday
5738	Tuesday	Tuesday	5773	Sunday	Sunday
5739	Monday	Monday	5774	Thursday	Thursday
5740	Friday	Friday	5775	Wednesday	Wednesday

Appendix 2
χ² (Chi-Square) Test of Significance

The Chi-Squared Goodness of Fit Test is used to determine whether there is a significant difference between the expected frequencies and the observed frequencies in one or more categories. In this case we are starting with the hypothesis that the true conjunction is Uniformly Distributed across the days of the week and testing to see whether the sample is consistent with this assumption. To test this hypothesis we compare the actual data to the expected values based on the hypothesis (i.e., all days being equally likely — thus for 400 trials each day is expected to occur 400/7 times) as follows:

χ² (Chi-Square) Test for Uniformity for 1700–2099

	Actual	Expected	(A-E)²/E
Sunday	67	57.1	1.700357143
Monday	57	57.1	0.000357143
Tuesday	52	57.1	0.462857143
Wednesday	67	57.1	1.700357143
Thursday	44	57.1	3.022857143
Friday	69	57.1	2.460357143
Saturday	44	57.1	3.022857143
Total	400	400	12.37
		p- value	0.054

The bottom row indicates there is less than 6% chance that the data is in fact Uniformly Distributed.