### A Short History of the Jewish Fixed Calendar: The Origin of the Molad

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#### I. Introduction.

It was always believed that the transition from the observation to the fixed calendar was clear-cut, with the fixed calendar immediately adopting its definitive form in 358/359, at the date of the inception. Indeed according to a tradition<sup>1</sup> quoted in the name of R' Hai Gaon,<sup>2</sup> the present Jewish calendar was introduced by the patriarch Hillel II in the Jewish Year 4119 AM (*anno mundi*, from creation), 358/359 CE.

The only discordant element with regard to this theory that the calendar adopted immediately its definitive form, was the fact that we find already in the Talmud that the postponement of Rosh Hashanah from Sunday was a later enactment.<sup>3</sup> Only some rare rabbinic authorities already recognized the later character of this postponement.

Indeed a passage of the epistle of R' Sherira Gaon implying that Rosh Hashanah of the year 505 C.E. was still on Sunday was generally considered as the result of a copyist mistake.<sup>4</sup>

It is only in the first decade of the twentieth century that new evidence appeared after the discovery of new documents in the Cairo Geniza.

Sefer ha-Ibbur by R' Abraham bar Hiyya edited by Filipowski, London 1851, p. 97 quotes a responsum of R. Hai Gaon dated from 4752 AM = 992 C.E. reporting this tradition.

<sup>&</sup>lt;sup>2</sup> R. Hai Gaon (939-1038) was the last and the most prolific Gaon. He belonged to the Yeshiva of Pumbedita.

<sup>&</sup>lt;sup>3</sup> See B. Niddah 67b. It appears that in the time of R' Yemar (427-432) Rosh Hashanah could fall on Sunday. See Ajdler (1966): *Hilkhot Kiddush ha-Hodesh alpi ha-Rambam*, Sifriati 1996, p. 670 and 684.

<sup>&</sup>lt;sup>4</sup> See *Iggeret Rav Sherira Gaon* part III, chap.4, p. 85 in the edition of R' Aaron Heyman, London 1910.

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The former conviction that the Jewish calendar immediately assumed its definitive shape at the moment of its inception was shaken by two major discoveries:

- The discovery of letters attesting the existence of an important dispute between the Babylonian community led by R' Sa'adia Gaon and the Palestinian community lead by (Aaron?) ben Meir about the *keviyah*<sup>5</sup> of the years 4682, 4683 and 4684.<sup>6</sup>
- The discovery and the publication in 1922 of a document from the Cairo Geniza: a letter from a Babylonian *Resh Galutah*<sup>7</sup> showing that the *keviyah* of the year 4596 (835/836 C.E.) was different than in our present-day calendar and that the Babylonian community received its calendric information from Palestine.

This last discovery was especially important; it proved beyond any doubt that almost five hundred years after the inception of the fixed calendar of Hillel, the fixed calendar in its present-day form had still not yet been instituted.

These two important discoveries were at the origin of much speculation about the history of the Jewish calendar. This history remains mostly conjectural because of the weak number of available pieces of evidence. But one thing is certain: our modern calendar in its final form was definitively not instituted before 922-924, after the end of the R. Sa'adia/Ben Meir controversy.

Hayyim Jehiel Bornstein<sup>8</sup> (1845-1928) played a major role in the analysis of these documents and in their correct interpretation. Tzvi Hirsh

מחלקת רב סעדיה גאון ובן מאיר בקביעת שנות ד"א תרפ"ב-תרפ"ד, ספר היובל לכבוד מו"ה נחום סוקאלאוו, וורשא תרס"ד משפט הסמיכה וקורותיה, התקופה, ספר רביעי, תרע"ט סדרי זמנים והתפתחותם בישראל, התקופה, ספר שישי, תר"ף תאריכי ישראל, התקופה, ספר שמיני, תרפ"א תאריכי ישראל, התקופה, ספר תשיעי, תרפ"א חשבון שמיטין ויובלות, התקופה, ספר האחד-עשר, תרפ"א דברי ימי העיבור האחרונים, התקופה, ספר ארבע עשר וחמישה עשר, תרפ"ב דברי ימי העיבור האחרונים, התקופה, ספר ששה עשר, תרפ"ג עיבורי מי העיבור האחרונים, התקופה, ספר שמי עשר וחמישה עשר, תרפ"ג

<sup>&</sup>lt;sup>5</sup> The *keviyah* refers to the length of the year and the weekday on which the month of Tishrei starts. These two pieces of information determine the exact layout of the entire year. This will be discussed in greater detail later in the paper.

<sup>&</sup>lt;sup>6</sup> See Bornstein, H. J: *Mahaloket Rav Sa'adia Gaon u Ben Meir*, Warsaw 1904.

<sup>&</sup>lt;sup>7</sup> For the text of the letter of the *Resh Galuta* see note 98.

<sup>&</sup>lt;sup>8</sup> He is the author of the following papers, in connection with the problems of the Jewish calendar:

Jaffe 9 (1853-1927) also made important contributions in this field. In general he appears more as the associate of Bornstein but some of his conclusions are more elaborate and more definitive than those of Bornstein. Akavya (Avraham Aryeh Leib Yakobovits) (1882-1964) devoted many years of research on the Hebrew calendar. He edited Korot Heshbon ha-Ibbur, the book of Jaffe and studied the tombstones of Zoar, which were discovered from about 1940 onwards and revealed the great diversity of the Jewish calendar even after the institution of the rabbinic calendar and even in Palestine in places not remote from the rabbinic centers. Stern, a historian, surveyed again all the available historical elements and put them in perspective in his book "Calendar and Community."<sup>10</sup> He put special emphasis on the lack of unity of the Jewish calendar and its great diversity through all the Jewish communities of the Diaspora. Furthermore, when later, after the sixth century, the rabbinic calendar asserted itself, all the distant communities, except the Babylonian and other neighboring communities, certainly remained unaware of the keviyah adopted by the Palestinian academy and had to live according to parallel approximate calendars of their own. It is only after the end of the R. Sa'adia Gaon-Ben Meir dispute that the rules of the calendar and the Four Gates Table<sup>11</sup> became known to the entire Diaspora.

In the present paper we try to outline the history of the Jewish calendar from the time of its inception until the tenth century, when it reached its definitive form.

At this stage, when mentioning influential scholars who made significant contributions to the field of the Jewish calendar, we must also mention the role of pioneer of Hayyim Selig Slonimski<sup>12</sup> (1810-1904). Before

<sup>&</sup>lt;sup>9</sup> Tzvi Hirsh Jaffe was born in Russia on 11 Sivan 5613. He had a thorough Talmudic education. He was an autodidact mathematician and talented engineer and inventor of a calculating machine. He was the editor of Azaria de Rossi's book מאור עיניים Warsaw 1899. He wrote explanatory notes to the Hebrew translation by Shaeffer of the History of the Jews of Graetz. He wrote the article Ben Meir in the *American Encyclopedia Otzar Israel*. But his opus magnum is his book קורות השבון העיבור that was edited by Akavia in Tel Aviv 1931.

<sup>&</sup>lt;sup>10</sup> Sacha Stern, *Calendar and Community, History of the Jewish Calendar, Second Century* B.C.E.-Tenth Century C.E. (Oxford University Press, 2001).

<sup>&</sup>lt;sup>11</sup> Table discovered by the Babylonian *meabrim* (mathematicians and specialists of the Jewish calendar). It allows finding the *keviyah* of a given year in function of its rank in the cycle of 19 years and the Molad of that year, i.e. the Molad of the month of Tishrei, at the beginning of that year. See Appendix C.

<sup>&</sup>lt;sup>12</sup> Hebrew popular science writer, popularizer and inventor (he was awarded a prize by the Russian academy of Science in 1844 for a calculating machine).

the discovery of the documents of the Cairo Geniza, he had already discovered that the Jewish Molad is derived from the table of mean conjunctions of Ptolemy's Almagest.<sup>13</sup> Similarly he was the first to state the late character of the *tekufah* of **R'** Adda bar Ahava. This concept seems to be a Spanish invention of the tenth century.<sup>14</sup>

In order to describe the evolution of the Jewish fixed calendar we will examine thoroughly the tables constructed by Jaffe in order to reconstruct the Jewish calendar in its different stages of development and make the critical analysis of the assumptions on which they are built.

Jaffe was probably overconfident in his mathematical achievements. The aim of this paper is to show how Jaffe constructed his tables for the different stages of development of the Jewish calendar and to distinguish between established and more questionable facts.

In this manner, the main achievements of Jaffe in his book *Korot Heshbon ha-Ibbur* will be made available to the modern reader who has no access to both the papers of Bornstein and the more systematic but difficult book of Jaffe. Even if some of their conclusions may be contested, these works remain authoritative in many respects. This paper aims at paying them homage, especially to Jaffe, Talmudist, mathematician and historian of great value, closely bound to all the research and discoveries of Bornstein but forgotten and neglected. He was even forgotten by the editors of the *Encyclopedia Judaica*.

# II. The calendar of Hillel<sup>15</sup> from 359 until the beginning of the seventh century (about 648).

According to a tradition quoted in the name of R' Hai Gaon, the present Jewish calendar was introduced by the patriarch Hillel II in 4119 (358/359).

<sup>&</sup>lt;sup>13</sup> Yessodei ha-Ibbur, Zitomir 1865, pp. 49-51.

<sup>&</sup>lt;sup>14</sup> Yessodei ha-Ibbur, Zitomir 1865, pp. 43-45.

<sup>&</sup>lt;sup>15</sup> The name of Hillel II the Patriarch is associated with the calendar instituted in 358/359 C.E. according to the tradition reported in the responsum of R' Hai Gaon. However the name of Hillel II is not mentioned in the Talmud and it is not certain at all that he had a direct part in this calendar. Maimonides does not mention him and is probably not aware of the tradition reported by R' Hai Gaon. It is clear that Rabbi Yose or Yousa, always mentioned in the Jerusalem Talmud in connection with the rules of the calendar, must have had a preponderant part in the foundation of this calendar.

We already demonstrated<sup>16</sup> that a pre-calculated calendar was established by the Court of Tiberias and sent to Babylonia from about 325 onwards. This calendar, however, was still a semi-empirical calendar replicating a calendar based on the first visibility of the new moon. By contrast the calendar instituted in 359 seems to be a completely calculated calendar based on a mean conjunction called Molad. The basic assumptions of this calendar, according to Jaffe, were probably the following:

- The Molad of Nissan 4119 was chosen near to the moment of the maximum of the solar eclipse,<sup>17</sup> which occurred on Monday afternoon 15 March 359 C.E. exactly the day of the inception of the new calendar.<sup>18</sup>
- The lunation adopted in the new calendar was 29 days 12 hours 792 *halakim<sup>19</sup>* (written 29-12-792).<sup>20</sup> At this epoch they did not use the

<sup>20</sup> According to the statement of Ravina in B. *Arakhim* 9b: מתקיף לה רבינא, והאיכא יומא דשעי ויומא דתלתין שני.

יומא דשעי the day of hours. It corresponds to a day resulting of the accumulation during 3 years of the excess of the length of the Jewish month of 29 days 12h 40m with regard to 29 days 12h: 36 \* (2/3) = 24 hours = 1 day.

<sup>&</sup>lt;sup>16</sup> See Ajdler (1996): "Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam," *Sifriati* 1966, p. 693-697 and Ajdler (2004), "Rav Safra and the Second Festival day," *Tradition*, vol 38, no. 4, pp. 3 – 28.

<sup>&</sup>lt;sup>17</sup> Solar eclipses always occur at the true lunar conjunction. The Molad used in the calculation of the Jewish calendar is a mean conjunction and an approximation of the true conjunction. If the sun and the moon were moving in the same plane (apparent movement seen from the earth) we would have a solar eclipse at each lunar conjunction. In fact these planes are distinct and the solar eclipses occur rarely. Anyhow the solar eclipses always occur at a true lunar conjunction, near to a mean conjunction and a Molad.

<sup>&</sup>lt;sup>18</sup> It is noteworthy that the day of inception of the calendar, the Molad of the calendar coincided with the true conjunction.

<sup>&</sup>lt;sup>19</sup> 792 *halakim* is 44 minutes. The lunation currently used, 29 – 12 – 793, was probably adopted at the end of the eighth century. Indeed Rabbi Pinhas, a Palestinian liturgical poet mentioned the division of the hour in 1080 parts. This division of the hour was specifically designated for the lunation of 29 days 12 hours and 793 parts. It is not known to have been used in any other context. Similarly the same Rabbi Pinhas mentions in his composition *Kiddush Yerahim* the cycle of 19 years. This cycle is also mentioned at the end of chapter 8 of *Pirquei de R. Eliezer*. The lunation of 29 – 12 – 793 and the leap years in each 19 year cycle, 3-6-8-11-14-17-19, could thus have been adopted at about the end of the eighth century. See Stern (2001) p. 197 and p. 204.

*helek* and did not divide the hour into 1080 *halakim*. They satisfied themselves with the division of the hour in 15 *hayil*,<sup>21</sup> a *hayil* representing 4 minutes or 72 *halakim*. The length of the month was thus noted 29 - 12 - 11 (i.e., 29 days 12 hours and 11 *hayil*). The length of a month is thus 4 weeks and 1 - 12 - 11. We say that the remainder<sup>22</sup> of a month is 1 - 12 - 11. Similarly the remainder of 6 months is 2 - 4 - 6, the remainder of 12 months is 4 - 8 - 12 and the remainder of 13 months is 5 - 21 - 8.

- The rules of the calendar were about the same as today except that the first day of Rosh Hashanah may fall on Sunday. The rules were thus the following:
- The postponements were DU (Wednesday and Friday) and  $\pi$ ' or 18 hours (noon).<sup>23</sup>

יומא דתלתין שני is an additional day resulting from the accumulation during 30 years of the difference between 29d 12h 44m and 29d 12h 40m: 12 \* 30 \* 4m = 1440m = 1 day.

The length of the Jewish lunation was thus 29d 12h 44m = 29 - 12 - 792.

<sup>&</sup>lt;sup>21</sup> See Baraïta de Shemuel chap 2 and 3. 1 Hayil = 1° of the equator and therefore also 4 minutes. We also have 1 hayil = 72 halakim and 1 minute = 18 halakim. 11 hayil = 792 halakim.

<sup>&</sup>lt;sup>22</sup> With regard to the greatest multiple of 7 days included.

<sup>23</sup> I.e., Rosh Hashanah is declared on the day of the Molad except when the Molad is on Wednesday (D for *dalet* the 4th day of the week) or Friday (U for vav the 6th day of the week), or when the Molad is at noon or later on any of the other days. In these cases Rosh Hashanah is postponed to the next day, unless that day is Wednesday or Friday in which case it is postponed to the next day, thus in all two days). The noon cut-off point is called Molad Zaken. Our current calendar has ADU postponements, i.e., besides not allowing RH to be on Wednesday or Friday we also do not allow it to be on Sunday (A for aleph). The DU postponements have deep roots in the Talmud. Until the second half of the third century, Rosh Hashanah could fall on any day of the week and Yom Kippur could be on Friday and Sunday (this is possible only if Rosh Hashanah is on Wednesday or Friday); see references in the Mishnah: Shabbat XV; 3, Shabbat XV; 19, Menahot XI; 7 (see also Maimonides' commentary ad locum), Menahot XI; 9. The ancient tradition reported in Rabbi Eliezer's name in Vayikra Rabbah XXIX; 1, according to which the seven days of the creation began on Sunday, Elul 25 of the year 1 AMI (the first year of Beharad), belongs to this period. Indeed it implies that Tishrei 1 of the year 2 AMI (the second year of Beharad or the first year of Weyad) was on Friday. According to the rules of our present calendar, this is impossible, Tishrei 1 cannot be a Friday and Elul 25 cannot be a Sunday. According to our modern calendar Elul 25 was a Monday and Tishrei 1 was a Saturday. The postponement DU was introduced during the second half of the third century by Rabbi Johanan in the time of Ulla ben Ishmael, see B. Rosh

• The length of the year was:

For an **ordinary** year: 353, 354, or 355 days with the following designations:

- 353: A **defective** year, in Hebrew הסרה הסרה. Shift of successive RH, 3 days.
- 354: A **regular** year, in Hebrew כ for ככסדרן. Shift of successive RH, 4 days.
- 355: A full year, in Hebrew ש for שלמה. Shift of successive RH, 5 days.

For a leap year: 383 days, 384, or 385 days with similar designations:

- 383: A **defective** year, in Hebrew הסרה החסרה. Shift of successive R.H: 5 days.
- 384: A regular year, כסדרן Confirt of successive RH, 6 days.
- 385: A full year,  $\mathfrak{W}$  for שלמה Shift of successive RH, 7 = 0 days. (When the length of the year is a multiple of 7, successive RH are on the same day.
- The derivative postponements (resulting from the former rules) were then<sup>24</sup>:

1–9–3 (ג ט ג) in an ordinary year (בפשוטה), i.e., if the Molad is at or later than Sunday 3:12 am Rosh Hashanah is postponed to Monday.

2-15-8 (ב טו ד) in a year following a leap year (עיבור), i.e., if the Molad is at or later than Monday 9:32 am, Rosh Hashanah is postponed to Tuesday.

• There are 18 different types of years. Years that have the same starting day for Rosh Hodesh Tishrei and Nissan are said to have the same

Hashanah 20a and Y. Megilah I; 2. Our current calendar also does not allow Rosh Hashannah to be on Sunday (A for 1st day of the week) This postponement is also discussed in the Talmud; see B. Succah 43b, Y. Succah IV; 1. We can deduce from B. Niddah 67b that in the time of Rav Yemar, head of the academy of Sura (427-432) Rosh Hashanah could still fall on Sunday.

At times the application of the standard postponements can lead to years, which do not conform to the 3 possible year lengths for regular and leap years. These anomalies are rectified by the introduction of *derivative* postponements. See Appendix B for a discussion of these derivative postponements in the early calendar and in our current calendar.

*keviyot.* This *keviyah* was often designated by a triplet of letters,<sup>25</sup> e.g., געו where the first letter (1) designates the starting day of Tishrei (Shabbat), the second letter ( $\mathfrak{W}$ ) designates the length of the year (full) and the third letter ( $\mathfrak{k}$ ) designates the starting day of Nissan (Tuesday). The 6 possible year lengths and 5 possible weekdays of RH led to 18 types of years: 9 types of ordinary years and 9 types of leap years. Ordinary years: אכג, בשה, גכה, גשו, החו, הכז, השא, זחא, זשא (10 אכה, בחה, בכו, בשה, גכז, החה, השג, זחג, זשג

The cycle of intercalation of 19 years did not yet exist. The rule of intercalation or the rule of the equinox is that Pesah cannot fall before March 19.<sup>27</sup> However the years were classified in table x of Jaffe,<sup>28</sup> in groups of 19 years, according to the principle of a fictitious cycle of intercalation of 19 years allowing an easy examination of the leap years with regard to our cycle of intercalation today.

The table  $\aleph$  of Jaffe for the years 4119 (358/359) until 4408 (647/648) was constructed on the preceding assumptions. It gives for each year the *keviyah* and the date of the first day of Passover. The leap years were chosen in such a way that Nissan 15 is never before March 19.

Let us come back to these different assumptions. The solar eclipse of Monday, March 15, 359 was at 15h 54m <sup>29</sup>Jerusalem mean time (ancient

Instead of 14 types of *keviyot* in our present-day calendar. i.e. Regular : זישג, השא, בחג, גכה, זחא, בשה הכז Leap: השג, החא, השג, החא, זעשה, גכז, זחג,

<sup>&</sup>lt;sup>25</sup> The 3<sup>rd</sup> letter in the triplet while convenient is not necessary. It is automatic based on the first 2 letters. Note: Tur gives 2 day *Keviyot* while *Prei Hadash* gives the triplet. Note: In this system Pesah can start on Friday, in ours it cannot (because Rosh Hashanah would then be on Sunday).

Indeed when we consider the six possible lengths of the year and the four possible weekdays of Rosh Hashanah, we find 14 different types of years.

For a complete table of these 14 calendars, including the distribution of the Shabbat's readings and *haftarot* see Akavya (1953) pp. 50 – 53, *Otzar Yisrael*, vol. 7, p. 310, Friedman (1971) pp. 218 – 219, Sar Shalom (1984) pp. 55 – 69, Slonimski (1852) pp. 50-, Slonimski (1865) pp. 59 -.

<sup>&</sup>lt;sup>27</sup> This questionable assumption of Jaffe will be discussed beneath.

<sup>&</sup>lt;sup>28</sup> See Korot Heshbon ha-Ibbur, Zevi Hirsh Jaffe, edited by Akavia, ha-Darom. Jerusalem 1931. The book is available at the New York public library \*ZP 735 Microfilm. See also the detailed appendices at <www.Hakirah.org/vol20Ajdler Appendices.pdf>.

<sup>&</sup>lt;sup>29</sup> This is 3:54 pm conventional time where the day begins at midnight.

style),<sup>30</sup> slightly less than the time calculated by Jaffe of about 6 p.m. Jaffe assumed that the Court fixed the epoch of the Molad at 18h. The epoch of the Molad was thus  $3 - 0 - 0.3^{11}$  Jaffe believed that the true conjunction was near to 6 pm, the conventional beginning of the night and therefore his assumption was genuine. This assumption is thus acceptable although 2 - 22 - 0 would have been more precise. We will see that Jaffe's assumption allows explaining and justifying different pieces of evidence, which could not be explained otherwise.

- 1. As the conjunction and the beginning of Nissan was on March 15, 359 Pesah was certainly after the spring equinox and therefore the rule of the equinox according to which, Pesah must be in the month of the spring, was respected for an ordinary year. The year 4119, corresponding to the fifteenth year of a fictitious cycle of 19 years was thus an ordinary year. It is thus easy to calculate the modern Molad of this month; we find 3-3-671 instead of 3-0-0 thus a difference of 3 hours  $671 halakim.^{32}$
- The most problematic aspect of the table x is Jaffe's assumption about the adopted rule of the equinox that Pesah cannot fall before March 19. Jaffe assumed that the rule of intercalation of the Jewish calendar was the rule of the equinox that Rabbi Huna bar Abin sent to Rava (Rosh Hashanah 21a):

שלח ליה רב הונא בר אבין לרבא: כד חזית דמשכא תקופת טבת עד שיתסר בניסן, עברה לההיא שתא ולא תחוש לה. When you see that the winter lasts until Nissan 16, intercalate that year and don't pay attention to any other sign of intercalation.

<sup>&</sup>lt;sup>30</sup> The eclipse of March 15, 359 on 1582256.14547 JD at 15h 29h 29s ET (See Mucke, H. and Meeus, J. Canon of Solar eclipses – 2003 to + 2526. Astronomisches Büro, Wien. 1983). The difference  $\Delta T = ET - UT \sim 1h 40$  m. Therefore the time of the eclipse was 15m 29m 29s – 1h 40m + 2h 21m = 16h 10m 29s Jerusalem modern mean time, and 15h 54m al-Battani Jerusalem mean time (ancient style of calibration of the mean time) (See Ajdler (2005): The Equation of Time in Ancient Jewish Astronomy, BDD 16, p. 14.) slightly before the time calculated by Jaffe of about 18h, probably using the tables of the Canon of Oppolzer, (Theodor Ritter von Oppolzer (Prague 1841 – Vienna 1886) : Canon Der Finsternisse, Vienna 1887).

 $<sup>31 \</sup>quad 3-0-0$  means the beginning of the third day, hence Monday at 6 p.m.

<sup>&</sup>lt;sup>32</sup> See Appendix D, I for the detail of the calculation.

According to modern scientific data, during the fourth century the true equinox was on March 20<sup>33</sup> and the mean equinox was on March 22.

According to the rule of the equinox of Rabbi Huna bar Abin Nissan 16 may fall on the day of the mean equinox, according to the understanding of R' Hananel<sup>34</sup> and R' Abraham bar Hiyya.<sup>35</sup> It may fall on the day following the mean equinox according to Rashi<sup>36</sup> and Rambam.<sup>37</sup> Thus according to the rule of the equinox, with the understanding of R' Abraham bar Hiyya, Nissan 16 could fall on March 22 and the first day of Passover could be on March 21. We know of effective cases of Pesah beginning on March 21.<sup>38</sup> Thus Pesah could begin on March 21 and the limit of March 19 adopted by Jaffe seems difficult to justify.

However the Christians considered that the true equinox is on March 21 and therefore, according to the rules adopted at the Council of Nicaea, Easter could fall the earliest on Sunday March 22. Indeed the rule of intercalation adopted by the council of Nicaea said: Easter is on Sunday following the fourteenth day of the moon, which reaches this stage on March 21 or slightly later.<sup>39</sup>

<sup>&</sup>lt;sup>33</sup> See J Meeus, Astronomical Tables of Sun, Moon and Planets, Willmann-Bell, 2<sup>nd</sup> edition 1995, pp. 109-110.

<sup>&</sup>lt;sup>34</sup> Commentary on B. Rosh Hashanah 21a.

<sup>&</sup>lt;sup>35</sup> Sefer ha-Ibbur, Ma'amar III, chap. 5; edition Filipowski 1846, p. 92.

<sup>&</sup>lt;sup>36</sup> Rashi on B. Sanhedrin 13b: and B. Rosh Hashanah 21a: In fact Rashi understands that R' Huna bar Abin requires that the *tekufat* Nissan falls the latest on Nissan 14. But if it were on Nissan 15 he would make the month of Adar full and the year would remain an ordinary year. If we transpose this in the modern fixed calendar, in which Adar of an ordinary year has always 29 days, this could be understood as the possibility of having the *tekufah* on Nissan 15. The reasoning of Tossafot is similar to that of Rashi but they require that the *tekufah* falls the latest on Nissan 15. If the *tekufah* were to fall on Nissan 16 they would make Adar full and they would behold an ordinary year. Therefore I consider that the position of Rashi can be compared to that of Rambam while the position of Tossafot could be compared to that of R. Abraham bar Hiyya and R. Hananel despite the formal differences.

<sup>&</sup>lt;sup>37</sup> Hilkhot Kiddush ha-Hodesh IV: 2.

<sup>&</sup>lt;sup>38</sup> From a piece of evidence mentioned beneath it appears that effectively Passover i.e. Nissan 15 could begin as early as March 21 and the eve of Passover, which the Christians called the "Pascha" could fall as soon as March 20. This was considered too early by the Christians, for whom Easter could not occur before March 22, the day following March 21 which they considered as the day of the true equinox.

<sup>&</sup>lt;sup>39</sup> Thus according to the rule of Nicaea, Nissan 14 was the earliest on March 21 and Easter is the earliest on Sunday March 22. By contrast, if the Jews considered the rule of the equinox according to the understanding of R' Abraham bar

At many occasions the Christians complained during the period of the second -fourth century and even later that the Jews did not respect the rule of the equinox and celebrated their festival of Passover too early. One must however be very cautious in the appreciation of these accusations. As noted by Stern, there was a great diversity among the Jewish communities, some following the rabbinic calendar, others not. Furthermore remote communities far from Palestine and Jewish rabbinic centers were not aware of the rabbinic calendar and could not follow it. It is important to note that when the Christians reproached the Jews about their early celebration of Passover they didn't take into account that the beginning of the festival, the night of the Seder, belongs to the next day. For them, in the Julian calendar, it belongs to the day before. Furthermore, the Christian writers confuse the ostentatious preparations of the feast on Nissan 14 and the public burning of the leaven, with the actual festival, which is more intimate and less spectacular. In all Christian sources the Jewish "Pascha" referred to Nissan 14, the day when the Passover sacrifice, if applicable, would have been prepared.<sup>40</sup> They may have considered that this day is the beginning of the festival. Jaffe mentioned two pieces of evidence about the alleged early celebration of Pesah by the Jews.

The first piece of evidence<sup>41</sup> is related to the year 387; it states that the Church of Alexandria, which considered that Easter cannot fall before Sunday 22 March because of the rule of the equinox adopted at the Council of Nicaea, reproached the Church of Rome, that they celebrated Easter on March 21, together with the Jews, before the limit accepted by the ecclesiastical rules.

A second piece of evidence<sup>42</sup> mentions that the Christian Church had adopted a cycle of intercalation of 84 years in 298 C.E. This cycle departed from the incorrect assumption that the true vernal equinox falls on March 18. Jaffe assumes that the rabbinical Court in Palestine accepted this true equinox and considered that March 20 is the mean equinox and accepted

Hiyya and consider the *tekufab* on March 22, Nissan 16 is the earliest on March 22 and Pesah which begins on Nissan 15, begins the earliest on March 21. In fact the Seder evening was even on March 20. The Christians considered that the Jews began Pesah too early and did not respect the rule of the equinox. We have a piece of evidence relating that the Jews began Pesah in 387 C.E. on March 21.

<sup>&</sup>lt;sup>40</sup> See Leviticus XXIII; 5.

<sup>&</sup>lt;sup>41</sup> L Ideler, *Handbuch der mathem*. Und technischen Chronologie, 2 vol. (Berlin 1825). Vol II, p. 255.

<sup>&</sup>lt;sup>42</sup> L Ideler. Vol II p. 232.

therefore that the first day of Passover falls on March 19.<sup>43</sup> Only if the first day of Passover falls on March 18 would they intercalate the year. Apparently this is the justification of the date of March 19 adopted by Jaffe in his tables, as the limit of Passover and this is the basis of the calculation of the leap years in his reconstructed calendar. It leads to empirical fictitious cycles of intercalation 3-6-8-11-14-16-19<sup>44</sup> or 3-6-8-11-14-17-19.<sup>45</sup> We will see that this assumption is unlikely; it is very problematic and must be considered with much reservation. The most probable empirical order of intercalation was the fictitious cycle of 19 years,  $3 - 5 - 8 - 11 - 14 - 16 - 19.^{46}$ 

3. The first piece of evidence mentioned by Jaffe concerned the year 387 C.E. Stern (2001) mentions other sources from which it appears that the year 387 C.E. was a very special year; it was the subject of many intense polemical debates. In the West it set the Alexandrians against the Romans, in the East it set the Alexandrians against the early Easter observers who followed the Eastern tradition of observing Easter "with the Jews". Besides the piece of evidence mentioned by Jaffe we know the third homily of John Chrysostom "against the Jews" which was delivered in Antioch early in 387 C.E. against the Jews observing Passover before the equinox and against the Christians following them. Similarly, the letter of Ambrose, bishop of Milan, from 387 C.E. was a pro-Alexandrian document and an attack against the Roman Church.

We examine in Appendix D, 2 the year 4147 AMI corresponding to 386/387. We prove that this year was not a leap year. Its Molad, according to Jaffe's assumptions, was 5 - 9 - 360. It corresponds perfectly to the Molad of Jaffe: 5 - 9 - 5 in his table  $\aleph$ .

The *keviyah* of the year 4147 was thus in the calendar of Hillel as it is also the case in our modern calendar: השא. Thus 1<sup>st</sup> day of Nissan was on Sunday, March, 7; the 15<sup>th</sup> of Nissan was on Sunday, the 21st of March 387, and the preparations of the festival and the public burning of the

<sup>&</sup>lt;sup>43</sup> According to the rule of the equinox of Rabbi Huna bar Abin as understood by R' Abraham bar Hiyya.

<sup>&</sup>lt;sup>44</sup> This is the order ascribed to *Hakhamim* in the Baraita of the order of intercalation.

<sup>&</sup>lt;sup>45</sup> This is the present order of intercalation; it is ascribed to Rabban Gamliel in the Baraita of the order of intercalation.

<sup>&</sup>lt;sup>46</sup> This is the order ascribed to Rabbi Eliezer in the Baraita of the order of intercalation.

leaven was exceptionally early, on Friday the 13<sup>th</sup> of Nissan<sup>47</sup> or March 19.

Our assumption that the year 4147, the fifth year of the fictitious cycle of 19 years was an ordinary year is thus perfectly justified as we see that Nissan 15 of this ordinary year fell on Sunday 21 March and satisfied the rule of the equinox.<sup>48</sup>

This historical piece of evidence gives us precious indications about the practical rule of the equinox used by the Court of Tiberias at the end of the fourth century, during the first decades of the Jewish calendar.

However, according to the Christian rules adopted at the Council of Nicaea, Easter must be on the Sunday following the 14<sup>th</sup> day of the moon which reaches this stage on March 21 or immediately after.

In 387, Nissan 14 was on Saturday, March 20, and for the Church, this lunation was not paschal because it fell before March 21. The year 387 was thus a limit case for the Christians. In fact it appears that even according to the Christian lunar tables the 14<sup>th</sup> day of the moon was even a day before on March 19.<sup>49</sup> Therefore this year must be intercalated in the Ecclesiastic calendar. The full moon of March 387 was not paschal and Easter must be delayed to the next lunation. Now the 14<sup>th</sup> day of the next lunation, according to the Christian tables, was on Sunday April 18 and Easter must then be celebrated on the following Sunday, on April 25.<sup>50</sup> The Roman Church could not accept such a late celebration of Easter. We see now that the year 387 was really exceptional. It is because of the exceptional lateness of the Alexandrian Easter that the date of Easter became in that year the object of such intense polemical debates.

Regardless, we see that the Jews celebrated Passover on March 21 in accordance with the Jewish rule of the Equinox, according to the understanding of R' Abraham bar Hiyya and R' Hananel of the rule of *Shitsar* given by Rabbi Huna bar Abin. Indeed Nissan16 was on March 22, the day of the *tekufah* or mean equinox.

4. The second piece of evidence given by Jaffe is related to the fact that the Church of Rome considered in its intercalation cycle that the true equinox is on March 18. It is likely that this data could have influenced the local Jewish community and its calculation of the intercalated

<sup>&</sup>lt;sup>47</sup> Because of the Shabbat.

<sup>&</sup>lt;sup>48</sup> The *tekufah* or mean equinox was on March 22 and the rule of the equinox of rabbi Huna bar Abin must be understood according to the understanding of R' Abraham bar Hiyya and R' Hananel.

<sup>&</sup>lt;sup>49</sup> Stern (2001) p. 144.

<sup>&</sup>lt;sup>50</sup> Thus 35 days later!

years but there is no reason that the Palestinian Court would have been influenced by the data used by the remote Church of Rome. The only undisputable data is that the Jews in the East celebrated Passover in 387 C.E. on March 21. If they celebrated Passover even on March 19, we would certainly have more polemical material extant. Apparently their early celebration of Passover on March 21<sup>51</sup> was enough to create intense disputes because it was a sufficient reason for the Christians to intercalate their ecclesiastic year. Now if the Court of Tiberias accepted an early Passover on March 19, in contradiction with the rule of the equinox of Rabbi Huna bar Abin and the other rules of the equinox defined in the Talmud,<sup>52</sup> the number of disputes would certainly have been much greater and the year 387 would not have been the most exemplary case of Jewish deviation. In summary, this second piece of evidence could apply to the Jews of Rome, distant from Palestine and the Court, but not to the Court of Tiberias.

In conclusion, table  $\aleph$  corresponding to the calendar of Hillel during 5. the period 4119- 4408 with the moladot and the keviyot of the different years is a tremendous work. However, it was built on the basis of a problematic<sup>53</sup> assumption that the limit of Passover was March 19. Therefore the sequence of the leap years is problematic and in consequence also the *moladot* and the *keviyot* of the years following the problematic and critical years. At the inception of the calendar of Hillel the limit of March 21 for Passover seems the most likely. It would be generally associated with the orders of intercalation 3-5-8-11-14-16-1954 and 3-6-8-11-14-16-19.55 However we know that the Julian calendar has an excess of 1 day in 128 years with regard to the length of the tropical year and it is therefore likely that the accepted limit of Passover of March 21 moved back with the time to March 20, March 19 and probably March 18 at the end of the eighth century. It appears therefore that it is impossible to establish a fixed table reconstituting the Jewish calendar because there remain too many unknowns. These considerations are also valid for all the tables of Jaffe whose purpose

<sup>&</sup>lt;sup>51</sup> Preceded by the burning of the leaven on March 19.

<sup>&</sup>lt;sup>52</sup> See B. Sanhedrin 13b-14a.

<sup>&</sup>lt;sup>53</sup> And probably erroneous.

<sup>&</sup>lt;sup>54</sup> This is the order of intercalation of Rabbi Eliezer in the Baraita of the order of the leap years in the cycle of 19 years quoted in *Sefer Yessod Olam*, book IV chap 2. It would correspond to the oldest order of intercalation.

<sup>&</sup>lt;sup>55</sup> This is the order of intercalation of *Hakhamim* in the same Baraita.

is the reconstitution of the Jewish calendar between 359 C.E. and 838 C.E.

- 6. Another factor of uncertainty in the tables of Jaffe is a problem raised by Bornstein<sup>56</sup> and Jaffe:<sup>57</sup> did the ancient masters of the calendar take into account, at a moment of history, the *Molad Zaken<sup>58</sup>* in months other than Tishrei?<sup>59</sup>
- 7. The origin of this problem is the discovery, by these scholars, in Sefer ha-Pardes<sup>60</sup> of the school of Rashi and in the tractate Soferim, of elements about an unknown keviyah XWX, for a leap year. It could only be the reminiscence of an ancient keviyah no longer in use. Jaffe noted in his tables the years which would have been affected by this problem. The problem of Molad Zaken in Shevat and possibly in Kislev is an intricate problem which will be examined in <www.Ha kirah.org/vol20AjdlerAppendices.pdf>.

In order to examine the merits of the Table of Jaffe, despite the weak point mentioned above, let us examine other pieces of evidence mentioned by Jaffe.

• The date of the death of R. Ahai bar R. Huna on Sunday 4 Adar 4266 AMI.<sup>61</sup> It implies that the next year 4267 began on Sunday. This was the eleventh year of the fictitious cycle 224 of 19 years. It is likely that it was a leap year. In our modern calendar the Molad of 4267 is

<sup>&</sup>lt;sup>56</sup> *Ha-Tekufah* vol 16, 1923, pp. 270-273.

<sup>57</sup> Korot Heshbon ha-Ibbur, Tel Aviv 1931, pp. 168-172.

<sup>&</sup>lt;sup>58</sup> The Molad is *Zaken*, when it falls, on a permissible day for Rosh Hashanah, at noon or after. Then 1 Tishrei is delayed to the next permissible day.

<sup>&</sup>lt;sup>59</sup> In the months of Tishrei and Nissan, days and nights are approximately 12 hours long and Jewish civil days begin at sunset, close to 6 p.m. This cannot be said for the other months. When days are > 12 h then nights are < 12 hours. This, however, is of no practical consequence because according to the rules of the Jewish calendar, we consider the situation as if we were at the equator. Thus we consider them as standard days with day = nights = 12 hours. The Jewish civil days begin at 6 p.m. and the molad is *Zaken* if it is at noon of a permissible day for Rosh Hashanah or later. Tishrei 1 is then delayed to the next permissible day.

<sup>&</sup>lt;sup>60</sup> Sefer ha-Pardes, edited by R' H.L. Ehrenreich, Budapest 1924, p. 340 lines 33-34: אם יבוא סוכות יום שלישי ויהיו מרחשון וכסליו שלימין, יהיו כ"ט שבתות ולא נצטרך לכפול See Appendix E, 3.

<sup>&</sup>lt;sup>61</sup> AMI refers to *Beharad*, as we do today and AMII refers to *Weyad*.

1 - 22 - 983. The Molad of Hillel was 1 - 17 - 648 corresponding exactly to the Molad given by Jaffe 1 - 17 - 9.62

We see that the modern Molad could not have fitted because it introduces a *Molad Zaken* and Rosh Hashanah would have been postponed to Tuesday. The Molad calculated according to the assumptions of Jaffe explains that we just avoided the postponement of *Molad Zaken*,<sup>63</sup> and that Rosh Hashanah and Adar 4 were on Sunday.

Jaffe mentioned a reference<sup>64</sup> from the Christian writer Victorius ac-• cording to which in 590 C.E. Passover, Nissan 15, fell on Sunday March 26 together with the Christian Easter and indeed some churches celebrated Easter on that day.65 However, the Alexandrian Church, to which the writer belonged, decided to celebrate Easter on the 22<sup>nd</sup> day of the lunar month, on the next Sunday,<sup>66</sup>in order not to celebrate Easter together with the Jews. Let us check this situation and check if it was indeed an exceptional case. We saw already in other examples how the calculations must be performed, allowing the checking of Jaffe's tables.<sup>67</sup> We can calculate the following table for the year 4350 AMI and for following years, which seem to also have Pesah beginning on Sunday. We note that in our modern calendar Pesah falls on the Sunday of Easter in 4350, 4354 and 4374. However when we check the situation according to the Calendar of Hillel, there is a coincidence only in 4350, with Pesah occurring on Saturday in the two other years. Similarly if we examine the calendar of Hillel, we note that Pesah falls on Sunday in the years 4350, 4353, 4357 and 4377.

<sup>&</sup>lt;sup>62</sup> See Appendix D, 3.

<sup>&</sup>lt;sup>63</sup> Stern (2001) p. 182 note 113 wants to prove that the Molad was already the modern Molad but *Molad Zaken* was not yet observed. He ascertains even that in 836 C.E. (see the letter of the *Resh Galuta*) the *Molad Zaken* was not yet applied. This position seems indefensible. It seems unconceivable that the rules of the calendar would still have changed in 836 C.E. and that a new postponement, would have been introduced. I have always championed the principle that the rules of the calendar were introduced at its inception; only the postponement A was introduced later but it was already debated at the origin. Only technical elements subject to new observation or measurement could be adapted: the Molad, the length of the Jewish lunation or the date of the *tekufab*.

<sup>&</sup>lt;sup>64</sup> Ideler II, p. 264.

<sup>&</sup>lt;sup>65</sup> Together with the Jews.

<sup>&</sup>lt;sup>66</sup> Sunday 2 April 590.

<sup>&</sup>lt;sup>67</sup> The *keviyah* is deduced from the Molad, using the Four Gates Table (see appendix C). The date of Easter was calculated using the algorithm of the Julian Easter by Spencer Jones p. 69 in Astronomical Algorithms Jean Meeus; Willmann-Bell 1991.

However Pesah coincided with Easter only in 4350. The piece of evidence of the Christian writer Victorius, seems to indicate that the year 590 was an exceptional year with the coincidence of Passover and Easter. Our modern calendar cannot explain this exceptional character because the same coincidence should have occurred also in 4354 and 4374. The calendar of Hillel, based on the assumptions of Jaffe, gives a satisfactory explanation:

Table 1: Pesah and Easter on Sunday in 590 CE and the following years. An asterisk (\*) indicates a leap year, and a *keviyah* in bold character indicates that Pesah was the same day as Easter.

Years	Modern Calendar			Calendar of Hillel			Easter
	Molad	Kevi-	Pessah	Molad	Kevi-	Pessah	Sunday
		yah			yah		
4350	5-20-1074	זחא	26/3/590	5-14-11	השא	26/3/590	26/3/590
4353	6-12-175	זשג	24/3/593	6-5-13	זחא	22/3/593	29/3/593
4354*	3-20-1051	החא	11/4/594	3-14-10	גכז	10/4/594	11/4/594
4357*	4-12-152	השג	9/4/597	4-5-12	החא	7/4/597	14/4/597
4374	5-11-75	השא	31/3/614	5-4-8	הכז	30/3/614	31/3/614
4377	6-2-256	זחא	27/3/617	5-19-10	זחא	27/3/617	3/4/617

Pesah and Easter coincided only in 4530. In 4353, 4357 and 4377 Nissan 14 was on a Saturday later than March 21 and Easter could have been on the next Sunday, together with the Jews. However it seems that the Ecclesiastic lunar calendar was slightly different than the Jewish lunar calendar and, in these three cases, the fourteenth day of the moon was a day later, on the Sunday, delaying automatically Easter to the next Sunday.<sup>68</sup> It is of interest to note that the Christians created scandals when the Jews celebrated their festival "too early" before them but yet they felt obliged to delay Easter when both festivals coincided.

We see again that the modern calendar and the modern Molad cannot explain why the coincidence of Passover and Easter in 590 C.E. was such a particular event. By contrast the assumptions of Jaffe explain that this coincidence was unique.

#### Conclusion

The table  $\aleph$  of Jaffe is related to the period 359 – 648 C.E. The amount of evidence related to this period is not large but, nevertheless it is not negligible and greater than for any other period. The assumptions of Jaffe,

<sup>&</sup>lt;sup>68</sup> The algorithm of Spencer Jones takes these situations into account.

about the limit of Passover, are questionable and, even untenable and the order of the leap years is at times questionable.

Therefore, in the present paper we always try to verify any data and we do not rely on Jaffe's table. The examination of different pieces of evidence shows that the assumptions of Jaffe about the epoch of the molad and the length of the Jewish month give interesting results and explain many historical facts that would otherwise not be understandable. It is, however, necessary to be cautious and question the order of intercalation. We can finally say that his table is reliable except for years with Pesah (Nissan 15) before March 21, which raise a problem. For such a year we must delay Pesah a month and make it a leap year ending a month later. The next year then begins a month later and becomes an ordinary year. The *keviyah* of both years must be adapted using the Four Gates Table.<sup>69</sup> This makes it necessary to adopt a likely date for the limit of Passover and then adapt the table of Jaffe for the problematic years.

As a result of the date of the true equinox, the theoretical acceptable limit date for the beginning of Passover should be:

From about 300 until about 430, the limit of Passover<sup>70</sup> should be March 21.

From about 430 until about 560, the limit of Passover should be March 20.

From about 560 until about 690, the limit of Passover should be March 19.

From about 690 until about 820, the limit of Passover should be March 18.

This table is of course purely theoretical. However the ancients did not know the length of the tropical year and the date of the equinox with precision and we don't know at which rate they moved back the limit of Passover.

The Tables of Jaffe inform the reader about the civil date of Passover and allows changing the order of intercalation without too much difficulty.

<sup>&</sup>lt;sup>69</sup> See Appendix C.

<sup>&</sup>lt;sup>70</sup> Nissan 15.

#### III The Jewish Calendar from about 648 until 776. The Introduction of the postponement "lo ADU Rosh" in the seventh Century.

We have seen that Rosh Hashanah could fall on Sunday in the calendar of Hillel. We found evidence in the Talmud that in the beginning of the fifth century under the reign of Rav Yeimar, Rosh Hashanah could still fall on Sunday.<sup>71</sup>

In the epistle of Rav Sherira Gaon it mentions that R' Ahai bar R' Huna died on Sunday 4 Adar 817 of the era of the contracts<sup>72</sup> or 4266 AMI of *Beharad*.<sup>73</sup> This implies that 14 Adar (Purim) would have been on Wednesday, the following Passover on Friday and the following Rosh Hashanah on Sunday.

In the *Sheiltot*<sup>74</sup> of R' Ahai Gaon<sup>75</sup> the postponement A seems already old history and is presented at the same level as the two former postponements **DU**. For this reason Jaffe and Bornstein considered that the postponement **A** must have been introduced during the first half of the seventh century. Stern (2001) also refers to an additional reference, the *Sefer ha-Ma'asim*.<sup>76</sup> In this work reference is also made to Rosh Hashanah occurring on Sunday.

Jaffe constructed the table  $\aleph$  until 4408 and the table  $\beth$ , related to the second period with the postponement A from 4390 onwards. This places the introduction of this postponement between 629 and 648 C.E. This last date seems to fit all the extant pieces of evidence.

The rules of the calendar were thus the same as before except the additional postponement A. There was probably not yet a regular cycle of intercalation; the intercalations were probably calculated on the basis of

<sup>&</sup>lt;sup>71</sup> See note 3 above.

<sup>&</sup>lt;sup>72</sup> Also the Seleucid era.

<sup>&</sup>lt;sup>73</sup> The relation between these two eras is: 1 SE = 3450 AMI

<sup>&</sup>lt;sup>74</sup> Sheiltot of Rav Ahai, chapter 79. This work was completed after R' Ahai Gaon settled in Palestine, in about 750 C.E.

<sup>&</sup>lt;sup>75</sup> R' Ahai of Shabha (680-752) is generally called R' Ahai Gaon although he never was Gaon. When a vacancy occurred in the geonate of Pumbedita in 748, the exilarch named a pupil of R' Ahai as Gaon. Incensed at this slight R' Ahai left Babylonia and settled in Palestine where he ended his masterpiece the *Sheiltot*.

<sup>&</sup>lt;sup>76</sup> The Sefer ha-Ma'asim li-benei Yisrael, Hillel Newman, Yad Ben Tsvi, is a book of halakhot of Palestinian composition; the date of composition is uncertain but the first half of the seventh century is likely. See Stern (2001) p. 184.

an adopted limit for Passover which was adapted according to the acquired knowledge about the length of the solar year and the date of the equinox.

The basic assumptions of Jaffe for the calendar in that period were thus the following:

- The Molad had been chosen near the moment of the maximum solar eclipse, which occurred on March 15, 359 C.E. exactly the day of the inception of the calendar. This Molad was still valid.
- The lunation was still 29 d 12 h 792 *hal* or 29d 12h 44m. At this epoch they did not yet use the *helek*<sup>77</sup> and did not divide the hour into 1080 *halakim*. They could suffice themselves with the division of the hour in 15 *hayil*<sup>78</sup> a *hayil* representing 4 minutes or 72 *hal*. The length of the month was thus noted 29 12 11. The remainder of a month was 1 12 11, the remainder of 6 months was 2 4 6, the remainder of 12 months was 4 8 12 and the remainder of 13 months was 5 21 8.
- The rules of the calendar were about the same as today and the postponements were now the same as today. The rules were thus the following:
- The postponements were ADU (Sunday, Wednesday and Friday) and
   n' or 18 hours (noon).
- The length of the year for an **ordinary** year:
  - 353 days for a **defective** year. Shift of RH, 3 days.
  - 354 days for a regular year. Shift of RH, 4 days.
  - 355 days for an abundant year. Shift of RH, 5 days.
  - The length of the year for a **leap** year:
    - 383 days for a **defective** year. Shift of RH, 5 days.
    - 384 days for a regular year. Shift of RH, 6 days.
    - 385 days for an **abundant** year. Shift of RH, 7 = 0 days.

Indeed when the number of days of the year is a multiple of 7, the day of RH has no shift and remains unchanged.

- The derivate postponements (resulting from the former rules) are then:<sup>79</sup>
  - 3-9-3 in an ordinary year or ג ט ג בפשוטה.
  - 2 15 8 in a year following a leap year or נ טו ה אהר עיבור.

<sup>&</sup>lt;sup>77</sup> 1 minute = 18 *halakim*.

<sup>&</sup>lt;sup>78</sup> 1 *helek* = 4 minutes and 1 *Hayil* = 72 *halakim*.

<sup>&</sup>lt;sup>79</sup> See Appendix B.

Because of the introduction of the postponement A, the number of possible *keviyot* was reduced to 14 as today and the possible *keviyot* were the same as today:
 For ordinary years יבחג, בשה, גכה, הכז, השא, זחא, זשג זשג;

and for leap years בחה, בשז, גכז, החא, השג, זחג זשה.

The weak point of the table  $\supseteq$  of Jaffe is again the list of the intercalated years. However the limit of Passover of March 19 seems suitable during the period 560 – 690. It appears that from about 690 onwards the limit of Passover should have been March 18. By contrast with the first period, no piece of evidence could be produced.

### IV The Observation of September 776 C.E and the adaptation of the Molad.

Chapter V of the *Baraita of Samuel*, in our printed version,<sup>80</sup> begins as follows:

בשנת ארבעת אלפים וחמש מאות ושלשים ושש שוו חמה ולבנה שמטות ותקופות ולא נשתיר לחמה אלא שעה אחת בלבד<sup>81</sup>. מן ארבעת אלפים ותקל"ו שנה<sup>82</sup> ואילך שנה ראשונה מולד לבנה בתשרי בתחילת ליל ד', בניסן בליל ו' בשתי שעות גדולות. הרוצה לידע מולד לבנה יחשוב מששוו חמה ולבנה כמה שנים ויתן ד' ימים וארבע שעות גדולות לכל שנה ושנה.

<sup>&</sup>lt;sup>80</sup> As it appears in *Sefer Poel ha-Shem* with a commentary of R. Arieh Leib Lipkin based on the edition by R' Nathan Amram, Salonika 1861.

<sup>&</sup>lt;sup>81</sup> It speaks of a "great hour" equal to two hours.

<sup>&</sup>lt;sup>82</sup> The year 4536 mentioned in the *Baraita* of Samuel is counted according to the style AMII (*Weyad*) and it corresponds to 4537 AMI (*Beharad*). This year is the first year of *shemitah* and the first year of the great cycle of 28 years. This is certainly an important piece of evidence in favor of the thesis of the Gaonim against Maimonides and against Rashi and Rosh: the *shemitah* is always on years multiple of 7 when counted in the style AMI from *Beharad*. We can assume that the members of the council of intercalation had made an observation of the equinox on Thursday September 19, 776 and deduced from it the mean equinox or *tekufah* on Tuesday, September 17, 776 at 16h Jerusalem time. This was a fairly good observation with a precision of about 7 hours with regard of modern calculations. We see that the ancient original text which assumes a primitive lunation of 29 - 12 - 720 (see below) was adapted in the manuscript used by R' Nathan Amram, in order to perpetuate this observation and the decisions of the Council of intercalation.

This text did not exist in the version of the Baraita of Samuel quoted by R' Abraham ibn Ezra<sup>83</sup> and R' Abraham bar Hiyya;<sup>84</sup> their chapter V began with:

## הרוצה לידע בכמה בשבת מולד לבנה נופלת יחשוב משנברא העולם עד <sup>85</sup>. עכשיו כמה שנים ויתן ארבעה ימים וארבע שעות גדולות לכל שנה ושנה

It appears that the Molad Tishrei 4537 was fixed on Tuesday, September 17, 776 at 6 p.m. or 4 - 0 - 0 and the *tekufah* of Tishrei which occurred at 3 - 22 - 0 was apparently delayed to 4 - 0 - 0 in order to create an epoch when *tekufah* and Molad coincided.<sup>86</sup> This coincidence fitted perfectly the biblical narrative of the creation of the luminaries on the fourth day.

The year 4537 is the 15<sup>th</sup> year of a fictitious cycle of 19 years; it is assumed to be an ordinary year.

The Molad of Hillel of Tishrei 4537 was 3 - 18 - 1008. It was corrected after the observation of September 776 to 4 - 0 - 0 by the addition of 5 - 72, thus 5 hours and 1/15. The modern value of the corresponding Molad is  $4 - 3 - 363.^{87}$ 

We ascertain that the consequence of the use of a lunation of 29 - 12 - 792 from the inception of the calendar brought an accumulated difference of 5108 hal = 4h 788 hal. It is not far from the correction of 5h 1/15 that was made by adopting the Molad of 4 - 0 - 0. We don't know how they found the new value of their mean conjunction, the Molad. Did they find it from a number of eclipses like Ptolemy or did they simply consider that the lunation of 29 - 12 - 793 is more correct and they simply added the accumulated difference and rounded the result off? Anyhow it seems that they adopted a new epoch for the Molad on Tishrei 4537, 4 - 0 - 0.

<sup>&</sup>lt;sup>83</sup> Commentary on Shemot XII: 2.

<sup>&</sup>lt;sup>84</sup> Sefer ha-Ibbur, p. 36 edition Filipowski, London 1851.

<sup>&</sup>lt;sup>85</sup> The remainder of 12 months is 4 – 8 – 876 in the modern calendar, the remainder 4 – 8 – 0 corresponds to a month of 29 – 12 – 720. The text of the Braita of Samuel seems to consider the more primitive value of the Jewish month of 29 – 12 – 720. See the dictum of Ravina in B. Arakhin 9b. See also Ajdler (2004): "Rav Safra and the Second Festival Day: Lesson about the evolution of the Jewish calendar," p.17, *Tradition* Vol 38, N° 4, Winter 2004.

<sup>&</sup>lt;sup>86</sup> This is reminiscent of a situation in our modern calendar during the year 1 AMI. The Molad Nissan 1 AMI was on 4 – 9 – 642 and the *tekufah* of Nissan was on 4 – 0 – 0, both *tekufah* of Samuel and of Rabbi Adda (exactly a week before). Rosh Hashanah 4537 was on Thursday September 19, 776. The mean conjunction (based on experimental observation) and the Molad were placed at 4 – 0 – 0 or Tuesday, September 17, 776 at 6 p.m.

<sup>&</sup>lt;sup>87</sup> See Appendix D, 4.

By contrast it is certain that the moment of the autumnal equinox must have been determined experimentally. They apparently found a true equinox on Thursday 19 September 776 at about 4 p.m. and deduced from it the mean equinox on Tuesday 17 September 776 at about 4 p.m. in Jerusalem.<sup>88</sup> The date of the equinox given in the *Baraita of Samuel* is the mean equinox. This is a proof that the equinox generally considered in the study of the Jewish calendar and in the rule of the equinox is always the mean equinox. This confirmed that Nissan 16 could fall on March 19 and Pesah could then be on March 18 at the end of the eighth century.

We observe also, from the text of the Baraita, that their counting of the Sabbatical years was the same as today, according to the counting of the Geonim mentioned by Rambam in his *Hibbur*, *Hilkhot Shemitah ve-Yovel* X; 6 the year 4536 AMI was a sabbatical year.

Jaffe has constructed table 7 of Moladot<sup>89</sup> from 4542 onwards, based on the results of the observation of September 776. He adopted the following assumptions:

The cycle of intercalation is now fixed; it is the cycle 3 – 6 – 8 – 11 – 14 – 17 – 19

This assumption makes sense.<sup>90</sup> The earliest mentions of the19-year cycle is in the end of chapter 8 of *Pirquei de-Rabbi Eliezer*, a work generally dated to the eighth or the ninth century. It is also mentioned in the liturgical poem *Kiddush Yerahim* of R' Pinhas which was written not earlier than the mid eighth century.<sup>91</sup>

The adoption of a fixed order of intercalation represents a considerable evolution in the solar regulation of the Jewish calendar. Instead of being obliged to be dependent on the *keviyot*, and compare Nissan 16 with a date of the *tekufah*, the rule of the equinox of Rabbi Huna bar Abin

<sup>&</sup>lt;sup>88</sup> The spring equinox occurs about 2 days before the mean equinox. This difference is practically exactly 2 days in Ptolemy's Almagest.

<sup>&</sup>lt;sup>89</sup> See Appendix H at <www.Hakirah. org/vol20AjdlerAppendices.pdf>.

<sup>&</sup>lt;sup>90</sup> But it could also have been introduced a little later. This new procedure represents an improvement and a simplification of the Jewish calendar. It is also the origin of the problems of the Jewish calendar. The adopted cycle fitted during the period 838 – 1160. Afterwards it will become the origin of an increasing discordance between the Jewish calendar and the solar year. The rule of intercalation or the rule of the equinox will not more correctly work because Pesah will begin later and later with regard to the spring equinox and Pesah will dwell outside the month of spring towards the summer.

<sup>&</sup>lt;sup>91</sup> Because there is mention in this liturgical poem of a fast commemorating the earthquake of January 748 C.E.

would depend now, after the introduction of a fixed order of intercalation, on the distance of the *tekufah* to the Molad of Nissan.<sup>92</sup> The rule of the equinox would simply imply that the vernal *tekufah* may not fall later than 16 days or 384 hours after the Molad of Nissan in the sixteenth year of the cycle, in which Pesah is the earliest.

This new procedure would be perfect if the length of the tropical year was exactly equal to the length of the mean Jewish year. In reality the Jewish year is longer than the tropical year and the Jewish year will shift toward the summer. In fact it appears that this cycle of intercalation was probably introduced several tens of years too early. Indeed the adopted *tekufah* on September 17, 776 at 18h corresponds to a true vernal equinox on 19 March 18 p.m. and a first day of Pesah or Nissan 15 on 18 March. We observe in table  $\pi$  that the introduction of the cycle of intercalation 3 -6-8-11-14-17-19 leads to a limit of Passover of 17 and 18 March. The date of March 17 is still too early for the first day of Passover.<sup>93</sup> Although there is no clear-cut limit it seems that this order of intercalation would have fit better during the period 838 – 1160. It was introduced a little too early.

The *tekufah* used at this stage is not yet the formal *tekufah* of Rabbi Adda bar Ahava but the mean equinox deduced from the observed astronomical true equinox.

 The Council adopted a cycle of 13 \* 19 = 247 years corresponding to a synodical lunation of 29 – 12 – 793 + 905 / (13 \* 235) = 29 – 12 – 793.2962.

This assumption rests on a minor clue, an allusion of Ibn Ezra about the relinquishment of the cycle of 247 years<sup>94</sup> also called עיגול דרב נחשון, which convinced Jaffe that this cycle was once in use. Indeed the *Iggul* of Rav Nahshon of 247 years =13 \* 235 months corresponds to a remainder of 6 - 23 - 175 = 7 days - 905 *hal*.

<sup>&</sup>lt;sup>92</sup> This principle was already proposed by R' Isaac Yisraeli in Yessod Olam, Ma'amar IV, chap 2, p. 4a and chap 4, p. 6a. Jaffe, in Korot (1931) p.112 adopted the same principle to explain the evolution of the understanding of the rule of the equinox. Loewinger in *Al ha-Sheminit*, Tel Aviv 1986, pp. 25-26 proposed to understand Rambam H.K.H. IV, 2 on the basis of this principle but the argument is questionable.

<sup>&</sup>lt;sup>93</sup> In other words the cycle 3 - 6 - 8 - 11 - 14 - 17 - 19 was introduced too early. <sup>94</sup> רמז לעגולת רמ"ז, הלוחות הראשונים אשר שברת, יישר כחך ששברת.

Ibn Ezra in *Sefer ha-Meorot*, Leiden 1496 and 1550; Rome 1544; Frankfort on the Main 1624. This reference was mentioned by Jaffe p. 159 and Bornstein *Makha-loket* p. 142. See also Jaffe p. 158 two references to *seder de rav Nahshon* and *iggul de rav Nahshon*.

The cycle of 247 years contains 3055 months. If a month had a length of 29 - 12 - 793 then 3055 months = 121,201,015 hal = [M(181440)] - 905 hal.

Thus introducing a regular cycle of 247 years gives a supplement of 905 *hal* for 3,055 months.

Jaffe built the table  $\overline{n}$  with the following assumptions. The Jewish month is still considered as (29 - 12 - 11) = (29 - 12 - 792), but after the first year and then after successively all the 4 and 5 years, he adds 1 *hayil*. With this procedure he adds the complete cycle of 55 *hayil* or 3960 *hal* corresponding to 3055 months \* 1 *hal* + 905 *hal*.

The procedure proposed by Jaffe is thus rigorously correct, but it the fruit of his inventive spirit and his ingenuity. There is not the least piece of evidence that this cycle was **really in use** and, if this was the case, it is not sure at all that it was implanted this way.

It is also possible that this cycle was only a working hypothesis, which was abandoned and never used. The length of the lunation would have been fixed from 4542 onwards to  $29 - 12 - 793.^{95}$  The difference has no practical consequences for us.

#### V The Letter of the *Resh Galuta* of 836 C.E

J. Mann discovered an exceptional document from the Cairo Geniza and published it in 1922.<sup>96</sup> This document was called the letter of the *Resh Galuta*,<sup>97</sup>because its author appeared to be a very important and authoritative personality.

This letter reveals that Passover (15 Nissan 4596) of the year 836 C.E. was due to occur on a Tuesday, March 21, 836 while according to the

<sup>&</sup>lt;sup>95</sup> It is generally accepted that only at the introduction of the Jewish month of 29 – 12 – 793 the necessity to introduce the *helek* (1/1080 of the hour) was felt. The first mention of the division of the hour in 1080 parts is made in a liturgical poem of Rabbi Pinhas. Similarly the earliest mention of the 19- year cycle of intercalation is made at the end of chap VIII of *Pirquei de-Rabbi Eliezer* (generally dated to the eighth or ninth century) and in the *Kiddush Yerahim* of Rabbi Pinhas. R' Pinhas is supposed to have lived in the late eighth or early ninth century. See Stern (2001) p. 197 and 204. R' Pinhas mentions in his *Kiddush Yerahim* the fast commemorating the earthquake of January 748 C.E. and wrote certainly after this date.

<sup>&</sup>lt;sup>96</sup> J. Mann, (1920-1922) "The Jews in Egypt and Palestine under the Fatimid Caliphs" 2 vols. London. See vol. 2. pp. 41-42.

<sup>&</sup>lt;sup>97</sup> The Babylonian Exilarch. There is indeed at the end of the letter an allusion on the authority of the letter's author.

present-day calendar, it should have occurred on Thursday, March 23, 836. According to the Exilarch the year must be defective in order to prevent the visibility of the new moon of Nissan before the first day of the month.

Today, however, we are not concerned about this problem and the Talmud accepted the case of a first visibility one day before the first day of the month or a day later.<sup>98</sup>

#### Table 2: The situation according to our modern calendar. Rosh Hashanah is on Saturday in both 4596 and 4597. 4596 is the 17<sup>th</sup> year of a cycle; it is a leap year איז of 385 days and 1 Nissan is on Thursday.

4596 AM1	835 C.E.	Tishrei 1	Nissan 1
385 days		Saturday, August 28 Molad (6)-22-660	
	836 C.E.		Thursday, March 23 Molad (3)-15-811 <i>Molad Zaken</i> if Molad >=(3)-13-642
4597 AM1		Saturday, Sept. 16 Molad (5)-20-169 <i>Molad Zaken</i>	

Table 3: The data According to the Letter of the Resh Galuta

4596 AM1	835 C.E.	Tishrei 1	Nissan 1
383 days		Saturday August 28	
	836 C.E.		Tuesday, March 21
4597 AM1		Thursday, Sept. 14	

The year 4596, the seventeenth year of a cycle of 19 years, was a leap year. According to the modern calendar it was a full year of 385 days of the type איז with Passover on Thursday, April 6. It appears from the letter of the *Resh Galuta* that in reality the year was defective of the type איז זעה and Passover was on Tuesday, April 4. The calendar was different than the present-day calendar. In order to go further we must examine the following passage of the letter:<sup>99</sup>

<sup>&</sup>lt;sup>98</sup> See B. Erakhim 9b. See J. Ajdler, *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, Jerusalem 1996, p. 221.

<sup>&</sup>lt;sup>99</sup> For a complete transcription of the letter of the Resh Galuta see:

<sup>1.</sup> J. Mann, note 96.

#### משום סיהרא דניסן דקא מתיליד **ביממא** דתלתא בשבא בארבע שעות

- Bornstein followed the reading of Mann and understood that the Molad of Nissan was on Tuesday at 4 Jewish hours: 3 4 0 in our notations, Monday at 10 p.m. about 12 hours before our modern Molad.<sup>100</sup> This explains that there was no *Molad Zaken* in Tishrei 4597 and therefore the year was defective.
- Jaffe did not read ארבע שעות but assumed ארבע ידות or ארבע דנקות. He understood that the Molad was 40 minutes in the morning thus a Molad 3 - 12 - 720, very near to the Molad used at that time after the adaptation of the Baraita of Samuel in 776. We understand now why Jaffe championed the Iggul of Rav Nahshon; it allowed the assumed Molad used by the Palestinians to coincide with the Molad mentioned by the Resh Galuta. This also explains why there was no Molad Zaken in Tishrei 4597 and the year was defective and had 383 days. Now according to this understanding of Jaffe, the Resh Galuta was aware of the effective Molad of 3 - 12 - 720 and the *keviah* sent from Palestine was correct and incontestable. Why was he then justifying the decision sent from Palestine and championing the unity of the communities of Israel as if he was facing opposition and objection against the keviyah sent from Palestine? In order to answer this question Jaffe must invoke the problem of Molad Zaken in Shevat.<sup>101</sup> The Molad of Shevat 4597 would indeed be  $(3 - 12 - 720)^{102} + (2 - 4 - 4)^{102} + (2 - 4)^{102$  $(438)^{103} + (6 - 2 - 1012)^{104} = 4 - 20 - 10$ : The problem of *Molad Zaken* in Shevat was in the news and the Palestinians decided not to pay attention to it and not delay Rosh Hashanah 4597 to Saturday because of it.
- Stern (2001) proved irrefutably that the reading is ארבע שעות. He understands that the Molad was at four hours in the morning thus the Molad was 3 16 0. This Molad was very near the modern Molad

<sup>2.</sup> H. J.Bornstein, Ha-Tekufah Vol 14-15, Warsaw 1922, p. 346.

<sup>3.</sup> M. Kasher, 1949, Torah Shelema XIII, p. 170.

<sup>4.</sup> R Sar Shalom, 1985, Shearim le-Luah ha-Ivri p. 27.

<sup>5.</sup> S Stern, 2001, *Calendar and Community* pp. 277-283 (with Xerox copy of the original).

<sup>&</sup>lt;sup>100</sup> Such a difference seems difficult to justify.

<sup>&</sup>lt;sup>101</sup> See Jaffe (1931) pp. 98-102. See also Appendix E at <www.Hakirah. org/vol20AjdlerAppendices.pdf>.

<sup>&</sup>lt;sup>102</sup> The assumed Molad of Nissan 4596.

<sup>&</sup>lt;sup>103</sup> The remaining of 6 months in order to get the Molad Tishrei 4597.

<sup>&</sup>lt;sup>104</sup> The remaining of 4 months in order to get the Molad of Shevat 4597.

3-15-811 and perhaps it was exactly the same but the *Resh Galuta* rounded it off. Thus the Molad was already the same as the modern Molad and the *Resh Galuta* knew this Molad. The question is then: why was this year defective? Stern answers that the postponement of *Molad Zaken* was not yet in observance.<sup>105</sup>

It must be noted that all these positions are untenable:

- Bornstein does not explain the aim of the letter of the *Resh Galuta*. Indeed this letter is certainly not a letter of announcement of the *kevi-yah* of the year 4596. It does not even mention that the year 4596 is a leap year. On the other hand he doesn't explain and justify the discrepancy of 12 hours with regard to the modern Molad.
- Jaffe founded his explanation and his elaborate theory on an incorrect reading.
- Stern understands that the Molad is the same as today but the rule of Molad Zaken did not yet exist. It would be introduced only in about 838 C.E. The position of Stern seems unacceptable for many reasons.
  - 1. It seems difficult to imagine that a rule like *Molad Zaken*, of which the origin is "as obscure as is its rational",<sup>106</sup> would have been introduced so late at a moment when it seems that the Babylonians could already have been associated with the calendar committee and without their objection. Furthermore we do not see a plausible motivation for such an innovation.
  - 2. It is certainly less problematic to keep the rules of the calendar and adapt the Molad according to the latest understanding of astronomy than to change the rules, which are sanctified by their age.
  - 3. If we consider<sup>107</sup> that the work of al-Kwarismi about the Jewish calendar was genuine, it would mean that in about 825, the rules of the calendar, including *Molad Zaken* were known, the only unknown elements were the epoch of the Molad and of the cycle of 19 years.<sup>108</sup>

<sup>&</sup>lt;sup>105</sup> Stern (2001) p. 196. Stern had already used the same argument in order to explain the *keviyab* of the year 4266, the year of the death of R. Ahai bar Rav Huna (see above). Again he assumed that the Molad in Tishrei 4266 was the same as today or very near to it and he explains that at this time the postponement of *Molad Zaken* was not yet in observance. See Stern (2001) p. 195.

<sup>&</sup>lt;sup>106</sup> Stern (2001) p. 195.

<sup>&</sup>lt;sup>107</sup> This is an assumption but there is no certitude. See Langermann (1987) and Sar Shalom (1988) in *Sinai* no. 106, pp.26-51.

<sup>&</sup>lt;sup>108</sup> See Stern (2001) p. 185.

- 4. The assumption of Stern that the present-day Molad was already the same in 836 and in 506 C.E. and even earlier is in contradiction with the theory that the Molad was derived from Ptolemy's Almagest in about 838 C.E. after the completion of an Arabic translation.<sup>109</sup>
- 5. Stern does not provide a plausible explanation of the purpose of the letter of the *Resh Galuta*. He does not explain the reason an objections was raised against the *keviyah* sent from Palestine.

Because of all these arguments I propose another explanation. It rests on the general theory of the evolution of the Molad of Jaffe but it deviates from his interpretation of the letter of the *Resh Galuta* and its purpose.

We assume that in Tishrei 776 C.E. the Molad was fixed at 4 - 0 - 0 according to the observation of the Baraita of Samuel and in March 836 the Molad was still based on the Molad of Tishrei 776 and was 3 - 12 - 448 (for a lunation of 29 - 12 - 793) or 3 - 12 - 680 (for a lunation of 29 - 12 - 793) or 3 - 12 - 680 (for a lunation of 29 - 12 - 793.2962, following the *iggul de Rav Nahshon* according to Jaffe's assumption. This value is very near to that calculated by Jaffe).<sup>110</sup>

This Molad of Nissan was thus certainly before the limit of 3 - 13 - 642 and therefore there was no *Molad Zaken* in the following month of Tishrei;<sup>111</sup> the leap year 4596 was a defective year of 383 days and Pesah was on Tuesday and not on Thursday.

Under the caliph al-Mamun (786-833) the son of the celebrated Harun al-Rashid (766-809) there was a cultural renaissance and the translation of Ptolemy's Almagest appeared in two versions; an older one by al-Hassan ibn Quraysh and another dated 827/828 by al-Hajjaj. This letter would be a piece of evidence of the first critics against the Palestinian authority. Some influential scholars had studied the new translation of the Almagest and had probably deduced from the table of mean conjunctions of the Almagest that the mean conjunction of Ptolemy of March 836 was  $3 - 14 - 1041^{112}$  in Alexandria and after transformation to Bagdad time it was indeed close to 3 - 16, corresponding to 10 a.m. or 4 hours in the morning as indicated in the letter of the *Resh Galuta*. They argued that the molad being about 3-16, there must be a *Molad Zaken* in Tishrei 4597 and

<sup>&</sup>lt;sup>109</sup> See Stern (2001) p. 209 about the death of R. Ahai bar R. Huna on Sunday 4 Adar 4266.

<sup>&</sup>lt;sup>110</sup> Thus the *Resh Galuta* knew already the Molad used by the Palestinian *meabrim*. For the justification of the calculations see Appendix D, 5.

<sup>&</sup>lt;sup>111</sup> (3-13-642) + (2-4-438) = 5-18 and we reach the limit of *Molad Zaken*.

<sup>&</sup>lt;sup>112</sup> The mean conjunction of Ptolemy in Alexandria is always the modern Molad – 850 *hal.* The modern Molad of Nissan 4596 was 3 – 15 – 811, therefore the conjunction of Ptolemy in Alexandria of Nissan 4596 was 3 –14 – 1041.

therefore the year 4596 should be an abundant year of 385 days and Passover should be on Thursday. These scholars contested thus the keviyah sent from Palestine on the basis of the data found in the Almagest, which had just been translated into Arabic. The scenario could have been the following: the Exilarch was not aware of the true Molad used by the calendric calculators or *meabrim* and accepted the Molad 3 – 16 proposed by his contradictors, the readers of the Almagest. He must advocate in favor of the Palestinian's keviyah and against those contradictors who contested the fixing of the year on the basis of the Ptolemaic conjunction. This allows an understanding as to why this letter advocated in favor of the Palestinian's decision and the primacy and the unity of the communities. This letter was thus not a letter announcing the keviyah to the communities; it was a letter advocating for the unity of the communities around the keviyah sent from Palestine. It is probable that concurrently the Exilarch expressed the view of his contradictors and his doubts to the Palestinians. It is likely that the Exilarch's intervention led to a common meeting in around 838 in the course of which the new Molad was adopted, in order to solve the contradiction between the Palestinian Molad and the Molad deduced from the Almagest.

In my opinion the rules of the calendar were already fixed long ago but the Molad was still the object of changes and adjusting. The postponement of *Molad Zaken* was, like the other postponements, old history. Except for postponement A, all the postponements already belonged to the calendar of Hillel at the inception of the fixed calendar.

#### VI Our Present Molad is derived from the Almagest.

At a period when the evolution of the Jewish calendar was not yet imagined, Hayyim Selig Slonimski<sup>113</sup> had already remarked on the dependence of our Molad on the table of mean conjunctions of the Almagest. Slonimski had remarked that the first conjunction of the table of Ptolemy corresponds to the conjunction of Nissan 3014.

The epoch of the Almagest is 1 Toth, year 1 of the Era of Nabonassar corresponding to Wednesday, February 26, 746 C.E. at noon.

<sup>&</sup>lt;sup>113</sup> See the bibliography at the end of the paper.

The first conjunction of the table of mean conjunctions<sup>114</sup> in Ptolemy's Almagest is 24 Toth; 44'17"<sup>115</sup> corresponding to Toth 24, 17h 42m 48s after noon<sup>116</sup> or Saturday, March 22 – 746 at 5h 42m 48s a.m. (after midnight) or 11h 770.40 *hal* in Jewish hours corresponding after rounding off to 7 – 11 – 770. Ptolemy's table gives also the distance of the common position of mean sun and mean moon, at the moment of the mean conjunction, from the sun's apogee. For this first mean conjunction this distance was: 288°; 38' 50". After addition of the sun's apogee of 65°; 30' we get the common mean longitude of 354°; 08' 50". This conjunction preceded thus slightly the equinox; it was thus certainly the mean conjunction of Nissan 3014.

Now if we calculate the modern Molad of Nissan  $3014^{117}$  we find that it was on 7 - 12 - 540,<sup>118</sup> thus Saturday at 6h 30m a.m. in round figures. Slonimski considered that this coincidence could not be a mere chance. He considered that our modern Molad was deduced from the Almagest by the addition of 850 *halakim*. It is a noticeable point that the number of lunations between the Molad *Weyad*<sup>119</sup> and the Molad of Nissan 3014 is

Ptolemy's Almagest, G.J. Toomer, London 1984, p.278. On page 275, in the text it calculated that the conjunction was 23; 44, 17 days after the epoch, which was noon of Toth 1 of the era of Nabonassar. The astronomical day began at noon. By contrast Ptolemy tabulated 24; 44, 17 with the meaning: the 24<sup>th</sup> day of Toth, 44, 17 after noon (the whole day being 60 parts, 44'17" represents 0.738055555 of a day of 24 hours, i.e. 17h 42m 48s). Apparently the convenience of this notation to the user became so obvious that he adopted it also in the Handy Tables. This is probably also the origin of the inclusive notation for dates adopted in the Jewish calendar. For example 6 - 12 - 540, the Molad of Nissan 3014 means Friday at 12h 540 *hal*. In many calculations it would be more convenient to use the exclusive and homogenous notation 5 - 12 - 540 giving the time elapsed since the beginning of the week at Sunday 0h but the custom of the *meabrim* is to use the inclusive notation and designate the beginning of the week by 1 - 0 -0 instead of 0 - 0 - 0 (after the beginning of the week).

<sup>&</sup>lt;sup>115</sup> 44' 17" represents a fraction of the day; the whole day is 60'. Thus 44' 17" represents 44/60 + 17/3600 = 0.73333 + 0.00472 = 0.738055 of a day= 17h 42m 48s after noon.

<sup>&</sup>lt;sup>116</sup> The day of the ancient astronomers began at noon. This practice was in use until the beginning of the nineteenth century.

<sup>&</sup>lt;sup>117</sup> In the Jewish proleptic calendar meaning the fictitious calendar extrapolated before its inception.

<sup>&</sup>lt;sup>118</sup> We note that (7 - 12 - 540) - (7 - 11 - 770) = 850 *halakim.* 

<sup>&</sup>lt;sup>119</sup> The Molad of Tishrei 2 AMI (Tishrei of the second year of the era of *Beharad*). It is also called *Molad Adam* by contrast with *Beharad* called *Molad Tohu*. Originally the Aera Mundi was counted from the second year; it was the Era of *Weyad*, 2

 $37260^{.120}$  It gives a shift of the Molad of 24300 hal = 22.5 hours and therefore the epoch or Molad of *Weyad*<sup>121</sup> was (7 - 12 - 540) - (0 - 22 - 540) = 6 - 14.

### VII The meeting ועד המאוחר between Palestinians and Babylonians in ca. 838<sup>122</sup> C.E.

Bornstein and Jaffe assumed that a meeting was held in Palestine with the participation of the Babylonian specialists.<sup>123</sup> Their participation could have been motivated by the fact that the Babylonians had provoked this meeting in order to debate about the discrepancies observed between the *keviyah* sent from Palestine on the basis of their Molad, and the *keviyah* deduced from the mean conjunction found in the Almagest. This was the beginning of the active participation of the Babylonians to the fixing of the calendar.

We have seen that the Council of intercalation adopted in 776 C.E. a new Molad; its epoch was 4 - 0 - 0, Tuesday, September 17, 776 at 6 p.m. The modern Molad of this month of Tishrei 4537 is 4 - 3 - 363; thus a difference of 3 - 363 = 3.3361 hours.

AMI or 1 AMII. AMI is the new style of *Beharad* and AMII is the ancient style of *Weyad*.

<sup>&</sup>lt;sup>120</sup> The number of lunations between *Beharad* and Nissan 3014 is  $235 \times 158 + 12 \times 7 + 13 \times 4 + 6 = 37272$ .

The number of lunations between Weyad and Nissan 3014 is then 37272 - 12 = 37260.

<sup>&</sup>lt;sup>121</sup> During a long period this Molad was the epoch of the Molad (Adam). Ibn Ezra, in his *Sefer ha-Ibbur* related this Molad to the Biblical passage in Deut XXIII, 13.

<sup>&</sup>lt;sup>122</sup> In fact the date of 838 is a pure assumption; it is shortly after the letter of the Resh Galuta and about 80 years before the dispute, which began in 921.

<sup>&</sup>lt;sup>123</sup> We have no real evidence of such a meeting. Bornstein and Jaffe based themselves on the contents of a letter addressed by the Babylonians to the Palestinians at the occasion of the R' Sa'adia-Ben Meir dispute, mentioning the existence of such a meeting which would have given to the Babylonian scholars all the elements allowing them to perform by themselves all the calendar calculations. See Jaffe *Korot* (1931) p.187 and Bornstein, *Makhaloket*, 1904, pp. 88-89. However such a meeting makes sense. We have seen that the letter of the *Resh* 

*Galuta* was probably a piece of evidence of the contestation against the *keviyah* sent from Palestine because it was in contradiction with the table of conjunctions of the Almagest. This problem justified a meeting with the Babylonians, the authors of the contestation. Besides, only such a meeting could explain how the Babylonians acquired the knowledge allowing them to make independent calculations of the *keviyah* and contradict the Palestinians at the occasion of the dispute between Ben Meir and Sa'adia Gaon in 922.

It is likely that the purpose of the meeting was to reform the molad to bring it in accordance with the Almagest, which was the authoritative reference. In fact we have no real piece of evidence proving the reality of this meeting and therefore no information about its decisions. However, from the elements of the dispute between R' Sa'adia Gaon and Ben Meir and from the different exchanges of letters between both parties which were found in the Cairo Geniza, Bornstein and Jaffe found an allusion to a common meeting some eighty years before and they deduced that the object of the dispute between both parties was a difference of 642 halakim between their Moladot. The Molad of the Palestinians was 642 halakim less than that of the Babylonians. It thus seems that they adapted at this meeting the Molad according to the table of the Almagest. However it seems that without paying too much attention to this point, they made the adaptation differently. They did not realize that this difference would bring in the future such a dispute. The Molad of the Almagest for Nissan 3014 was after rounding off, 7 - 11 - 770 in Alexandria. According to Ptolemy's Geography the difference of longitude between Alexandria and Jerusalem is 5°; 30' corresponding to 22m or 396 halakim. The Molad in Jerusalem was thus 7 - 12 - 86. The Babylonians added another 454 *hala*kim in order to get a rounded off number, 7 - 12 - 540, for the Molad of Nissan 3014, from which they deduced the epoch of the era of the creation, (7 - 12 - 540) - (0 - 22 - 540) = 6 - 14.124 By contrast the Palestinians subtracted the remainder of six months i.e. 2 - 4 - 438 from the

<sup>&</sup>lt;sup>124</sup> Nowadays we consider exclusively the era of *Beharad*. But before the eleventh century the era of the creation was counted from the second year, it was the era of *Weyad*. This era is already mentioned in the Talmud, Avoda Zara 9b. All the dates in the Talmud are expressed in AMII. In B. Avoda Zara 9b it writes: 403 years of the era of the Destruction = 4231 AMII.

Thus 1 Era of the destruction= 4231 - 402 = 3829 AMII = 3830 AMI = 70 C.E. It seems interesting at his point to give the chronology of the first year of *Beharad*. This era was probably introduced because it placed the epoch of this era at the beginning of a cycle of 19 years.

The *tekufah* of Samuel of Tishrei: the *tekufah* of Samuel was on 24 September at 3 a.m. The Molad *Beharad* 2 - 5 - 204 was on Sunday, October 6 - 3760 at 23h 11m 20s.

<sup>1</sup> Tishrei AMI was Monday, October 7 – 3760.

<sup>30</sup> Marheshvan 1 AMI was Thursday, December 5 - 3760.

<sup>30</sup> Kislev 1AMI was Saturday, January 4 – 3759.

The *tekufab* of Samuel of Nissan was on Wednesday 22 Adar at 0h i.e. 4 - 0 - 0 or Tuesday, March 25 - 3759 at 6 p.m.

<sup>26</sup> Adar 1 AMI was Sunday, March 30 - 3759.

<sup>29</sup> Adar 1 AMI was Wednesday, April 2 – 3759.

Molad of Nissan 3014: 7 - 12 - 86 and found 5 - 7 - 728 for the Molad of Tishrei 3014. They rounded off this Molad to 5 - 7 - 540 by subtracting 188 *halakim*. This led them to a rounded off Molad for Nissan of *Tohu*: (5 - 7 - 540) - (0 - 22 - 540) = 4 - 9 - 0.

Apparently the participants did not find an agreement for a common decision. Thus Palestinians and Babylonians left each other with different Moladot, the Babylonians added 454 *halakim* to the conjunction of Ptolemy while the Palestinians subtracted 188 *halakim*. The Palestinians, who considered themselves as the principal concerned, probably left the problem open in the hope that new observations would help solve it definitively.

Jaffe supposed that at the end of the ninth century the members of the Palestinian council of intercalation were made aware of the observations of al-Battani: the determination of the equinox of 19 September 882 and the observation of the lunar eclipse of 21 July 882.<sup>125</sup>

The observation of the equinox<sup>126</sup> confirmed to them that the observation of 776 was acceptable and it informed them that the limit of Passover of March 17 connected to the new system of a regular cycle of intercalation 3-6-8-11-14-17-19 was now acceptable and justified.<sup>127</sup>

Molad Weyad or 6 – 14 was on Friday, September 26 – 3759 at 8 a.m.

This observation justifies already Passover on March 17.

<sup>24</sup> Elul 1 AMI was Sunday, September 12 – 3759.

<sup>1</sup> Tishrei 2 AMI was Saturday, September 27 – 3759.

In *Vayikra Rabbah* XXIX, 1 it states that the creation began on Sunday 25 Elul; this seems in contradiction with our table giving Sunday 24 Elul. Apparently this passage is anterior to the rule **lo DU Rosh.** The 1 Tishrei 2 AMI was on Friday and therefore the preceding Sunday was the 25 Elul. Similarly the Sunday 26 Adar 1 AMI was in this ancient calendar Sunday 27 Adar, the year 1 AMI being an abundant year of 355 days. This day would be Sunday 25 Adar 1 AMI if this year of Tohu was a defective year of 353 days. But this is contrary to our calendar. In other words the ancient traditions placing the beginning of the creation of the world on Sunday 25 Elul or on Sunday 25 Adar are anterior to our calendar and don't agree with it.

<sup>&</sup>lt;sup>125</sup> Jaffe had apparently no access to the original treatise of al-Battani and knew these observations through secondary sources like the information provided by *Yessod Olam* of R' Isaac Israeli. See *Yessod Olam*, *ma'amar* IV, chap. 7, p. 12a for the lunar eclipse.

<sup>&</sup>lt;sup>126</sup> See al-Battani Vol 1 pp. 42 and 210. The equinox occurred on 19 September 1h 15m a.m. ar-Raquah or 18 September 22h 39m UT. The modern value is 23h 05m. This observation is considered as one, if not the most, exceptional astronomical observation of history.

<sup>&</sup>lt;sup>127</sup> The equinox "observed" by al-Battani was on September 19, at 1h 15m a.m. ar-Raquah or at about 0h 48m corresponding to a mean equinox on 17 September

The observation of the lunar eclipse<sup>128</sup> would have persuaded the members of the council of intercalation that the mean conjunctions preceded the mean conjunctions of Ptolemy and therefore the rounding off adopted by the Palestinians seemed justified to them by contrast to the rounding off adopted by the Babylonians. Apparently Palestinians did not inform Babylonians of these last developments.

The problem is that there is no proof that the members of the council of intercalation already knew the treatise of al-Battani. Furthermore the details of the observation of the lunar eclipse are insufficient to know the mean conjunction<sup>129</sup> and al-Battani is unlikely to have published his works before the beginning of the tenth century. It is, however, correct that the comparison of the table of conjunctions of the Almagest and that of al-Battani allows us to conclude that the mean conjunctions of al-Battani preceded those of Ptolemy by 31 minutes if we take into consideration the longitudes of ar-Raquah of 73°; 15' and Alexandria of 60°; 30'.<sup>130</sup>

Thus the astronomical treatise of al-Battani would arbiter in favor of the Palestinian position. But it is not sure that the Treatise of Astronomy of al-Battani was known by the Palestinian council of intercalation before the outbreak of the dispute. In any case it seems likely that the entire discussion between Palestinians and Babylonians about the Molad was forgotten and two concurrent and contradictory methods of calculation of the *keniyah* coexisted until the outbreak of the dispute in 922 C.E, without the protagonists remembering the origin of the discrepancy.

at about the same hour. The following vernal mean equinox was then on 18 March at about 4 p.m. Thus Nissan 16 may fall on March 18 and Passover may fall on March 17. The observation of al-Battani supported the cycle of intercalation adopted, 3-6-8-11-14-17-19. This is the meaning of the statement of R' Juda ha-Levi in *Sefer ha-Kuzari*, book IV, chap 29 that the *tekufah* of Adda is in agreement with the observation of al-Battani.

<sup>&</sup>lt;sup>128</sup> See al-Battani, Vol 1, pp. 57 and 230. The lunar eclipse was on Tuesday 23 July 883 at 8h 06m p.m. or Wednesday 15 Av 4643 in the beginning of the evening.

<sup>&</sup>lt;sup>129</sup> We know that the relative position of the two bodies, the sun and the moon, may vary  $1.9^{\circ} + 5.4^{\circ} = 7.3^{\circ}$  from their mean value near the conjunction. As the hourly motion of D, the elongation moon-sun is 0.51°, the maximum interval between the mean new moon and the true new moon is 14.3 hours. At the moment of the full moon the situation is similar between the true and the mean full moon.

<sup>&</sup>lt;sup>130</sup> The time difference between ar-Raquah and Alexandria is thus 51 minutes. However in the book of al-Battani, *Opus Astronomicum*, Vol 1, p.42, it writes in the main text that this difference is 40 minutes; this would reduce the difference to 20 minutes.

#### VIII The Dispute of R' Sa'adia Gaon and Ben Meir

On Hoshana Rabbah 921 C.E. The Palestinian Gaon Ben Meir or his son proclaimed on the Mount of Olives that the months of Marheshvan and Kislev of 4682 would be defective. As a result Passover 922 would fall on Sunday instead of the following Tuesday if the year had been made full. And in fact, in 922 the Jews of Palestine and probably the communities in Egypt celebrated Passover on Sunday, two days before the Jews of Babylonia. This split between the communities of Palestine and Babylonia caused considerable agitation throughout world Jewry. References to this event can be found in non-Jewish documents. The Syrian Elias of Nissibis<sup>131</sup> wrote that in the year 1232 of the Seleucid era<sup>132</sup> dissension broke out between the Jews of the West (Palestine) and those of the East (Babylonia) with regard to the calculation of their holiday. The Jews of the West celebrated Rosh Hashanah 4683 on a Tuesday and those of the East celebrated it on the next Thursday.133 Similarly the Karaite Sahal ben Mazliah<sup>134</sup> also referred this event and sought to prove from this controversy that the rabbinic calendar calculations were groundless. According to the Babylonian Molad, in Tishrei 4683 there was the postponement Gatrad and in Tishrei 4684 there was the postponement Yah, therefore the keviyah of the three years 4682, 4683 and 4684 were then: בחג, הכז, השג. By contrast, the Molad of the Palestinians was 642 hal less and there was no postponement in Tishrei 4683 and 4684 and the keviyah of the three years 4682, 4683 and 4684 were: זשג, גכה, החא. Furthermore the astronomical situation was exceptional on Rosh Hashanah 4683: the true conjunction occurred about 1.5 hours after sunset on Monday evening. The lunar latitude was about 5°, an exceptional fact, the moon was seen on Tuesday evening in Egypt, in Palestine and even in Babylonia.

<sup>&</sup>lt;sup>131</sup> See Bornstein, Divrei Yemei ha-Ibbur ha-Aharonim, ha-Tekufah, Vol 16, Warsaw 1923 pp. 237-238.

<sup>&</sup>lt;sup>132</sup> According to the Jewish *Minian Shtarot*: 1 SE = 3450 AMI and 1232 SE = 4681 AMI. See Rambam, *Hilkhot Kiddush ha-Hodesh* 11, 16 and *Hilkhot Shemitah ve-Yovel* 10, 4. However there were other methods of calculation of the Seleucid era differing by a year or differing by the epoch adopted in March instead of September. Here it seems that the date corresponds to 4682 AMI.

<sup>&</sup>lt;sup>133</sup> Otzar Israel, entry "Ben Meir," written by Jaffe.

<sup>&</sup>lt;sup>134</sup> See Bornstein, "Divrei Yemei ha-Ibbur ha-Aharonim," *ha-Tekufah*, Vol. 16, Warsaw 1923, p. 237.

Year	Year	Babylonian	Keviyah	Palestinian	Keviyah
		Molad		Molad	-
921-922	4682*	4 - 11 - 932	השג	4 – 11 –	החא
				290	
922-923	4683	3-9-441	הכז	3 - 8 - 879	גכה
923-924	4684	7 – 18 - 237	בחג	7 – 17 –	זשג
				675	

Table 4: The years 4682, 4683 and 4684 according to the Palestinians and the Babylonians

The vision of the new lunar crescent was thus one day before the first day of Rosh Hashanah adopted by the Babylonians. The Karaites, who sanctified the first day of Tishrei at the moment of the vision of the new moon, celebrated their Rosh Hashanah on Wednesday. This was also an exceptional event: never before had the Karaites celebrated Rosh Hashanah before the Babylonian Rabbis. This event made a great stir and agitation in Egypt and the pupils of Rabbi Sa'adia Gaon were distraught. The letters exchanged between them and Sa'adia Gaon were preserved in the Cairo Geniza.

The Palestinian community saw with this vision the proof of the correctness of the calculation of Ben Meir and his *keviyah*. The truth is that the Talmud accepts such an inevitable situation: it is possible that the new crescent is seen one day before the *Keviyah*.<sup>135</sup>

Maimonides wrote about this problematic first visibility of the lunar crescent one day before the *yom ha-keviyah*,<sup>136</sup>

ודבר זה הלכה למשה מסיני הוא, שבזמן שיש סנהדרין קובעין על פי הראייה ובזמן שאין שם סנהדרין קובעין על פי החשבון הזה שאנו מחשבין בו היום ואין נזקקים לראייה, אלא פעמים שיהיה יום שקובעין בו בחשבון זה הוא יום הראייה או קודם לו ביום או אחריו ביום, וזה שיהיה אחר הראייה ביום פלא הוא, ובארצות שהן למערב ארץ ישראל.

It is thus a Mosaic tradition from Sinai that in times when there was a (Palestinian) Sanhedrin, declaration of New Moon Days was based on visual observation, while in times when no Synedrium existed, this declaration was based on calculations such as we are using today and no attention was paid to observation of the new crescent. Rather the day established by calculation might well coincide with the day

<sup>&</sup>lt;sup>135</sup> B. Erakhim 9b. See J. Ajdler (1996): "Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam," *Sifriati* 1996, pp. 225-226. This passage has raised many difficulties.

<sup>&</sup>lt;sup>136</sup> Hilkhot Kiddush ha-Hodesh V; 2.

in which the new moon became visible, but it might sometimes be the day before it or the day after<sup>137</sup> it. The latter case, however, when the calculated New Moon Day happened to be the day after the new moon became visible, occurred only rarely,<sup>138</sup> and then in the countries west of Palestine.<sup>139</sup>

It would be better to understand that, according to Maimonides' statement, the first vision of the lunar crescent before the *yom ha-keviyah*, the first day of the month, is a very rare event. However in areas situated west to Israel, the possibility of an early vision of the lunar crescent before the *yom ha-keviyah*, the first day of the month, is less exceptional.<sup>140</sup>

<sup>&</sup>lt;sup>137</sup> It can in fact last until two and even three days later. This passage is contradicted by another difficult passage in HKH VII: 7-8; see J. Ajdler "Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam," *Sifriati* 1996, pp. 226-227. Jaffe in *Korot* (1931) p. 197 at the note on bottom already proposed to correct the text and wrote, « או ביומים או ביומים או ביומים או ביומים או ביומים או ביומים או ביומים. Ibn Ezra in his commentary on *Vayikra* XXIII, 3 writes also that it happens sometimes that in Tishrei the *keviyah* is on Thursday and the new moon is seen only on Friday evening.

<sup>&</sup>lt;sup>138</sup> We must probably understand that the visibility of the new moon before the *yom ha-keviyah* is exceptional, but in the countries west to Palestine it is less exceptional.

<sup>&</sup>lt;sup>139</sup> Translation of Solomon Gandz in *Sanctification of the New Moon*, Yale Judaica Series, Volume XI, pp. 22-23.

<sup>&</sup>lt;sup>140</sup> R' Raphael ha-Levi from Hanover writes in his book "כללי סוד העיבור" still in manuscript in Jews College library in London:

ובמדינות שהן למערב ארץ ישראל, פירוש במדינות שהן למערב ארץ ישראל יהיה עת הראייה מאוחר לעת הראייה שבארץ ישראל, ואז אפשר אם היה המולד ג יח, או ה יח, או ז יח, לראות הירח באותן המדינות בליל ד' והקביעה יהיה ביום ה', כי בהקביעה הן שווין, ולפי זה במדינות ההן אין הפלא כל כך גדול כמו שהוא בארץ ישראל, אף על פי שהראייה בחוץ לארץ אינה מועלת כי אם בארץ ישראל והוא לא כתב זה אלא לדמיונו.

About the exceptional character of this early vision one day before the *yom hakeviyah*, he adds :

והוא פלא כי צריך להיות מולד אמצעי בתשרי ב" גטרד" וצריך שנה פשוטה וגם צריך להיות הקיבוץ מוקדם לאמצעי כדי שיהיה ריוח בין רגע קיבוץ עד עת הראייה יתר מן כ"ד שעות, וצריך שיהיה רוחב הירח צפונית חמש מעלות, ודבר זה שיהיו כל התנאים הממהרים הראייה ביחד הוא פלא גדול ואפשר שלא המצא תמצא בחמש מאות שנים. וכבר יגעתי ומצאתי תאמין שחקרתי וחפשתי בחיפוש אחר חיפוש משנת ד' אלפים עד שנת ה' אלפים ליצירה ולא מצאתי רק במשל אחד בשנת ד"א תרפ"ג שהיה המולד אמצעי בתשרי ג ט תמא ונדחה לחמישי ונראה הירח בליל ד' והיה יום הראייה ביום ד' יום אחד מוקדם ליום הקביעה.

This passage, which is a quotation from an unpublished manuscript from R' Raphael ha-Levi from Hanover is an exceptional piece of evidence of his calculation abilities (and patience) and of the reliability of Maimonides' visibility criterion. Imagine that Raphael Hanover, who had not the least idea of the R' Saadia-Ben Meir dispute, discovered the critical year 4683, among thousand years,

The conclusion of the R' Saadia-Ben Meir controversy at the advantage of the Babylonians had a tremendous consequence at the level of the unity of the Jewish people. Before 922 C.E, the Jewish calendar was communicated by the Palestinian *Gaon* on an annual or multi-annual basis.

It appears that from about 838 onwards, the Babylonians were able to make their own calculations and during the period of about eighty years preceding 922 C.E. they always agreed with the *keviyah* sent from Palestine. However the remote communities in Europe and Africa were certainly not informed in time of the calendar data and were not able to keep the festivals at the same time as the two great centers of Palestine and Babylonia.

However, Spain and Kairouan, two centers having narrow bonds with Babylonia, were probably informed in time. Only after the end of the dispute, did the rules of the calendar and the Four Gates Table became universally known and only then was the complete unity of the Jewish communities of the Diaspora achieved in the celebration of their festivals.

A second consequence, not less important, of the supremacy of the Babylonian community, was that, parallel to the fact that the Jewish calendar became universally known, it became also definitively stiff and rigid. As long as the Babylonian community accepted the Palestinian *keviyab*,<sup>141</sup> the council of intercalation, acting with much secrecy, had the possibility to adapt and improve the calendar. From this time onwards, the Jewish communities could participate in the development and the study of the Jewish calendar. It seems that the custom to count the Jewish calendar according to the era of Tohu (*Beharad*), beginning the era with a year L+1,<sup>142</sup> following a leap year, at the beginning of a cycle of 19 years of the proleptic<sup>143</sup> Jewish calendar, instead of the era of the creation (*Weyad*) beginning the era with a year L – 1,<sup>144</sup> preceding a leap year, was introduced

in which exceptionally the new crescent was visible one day before the Babylonian *keviyah*.

Ibn Ezra noted in his commentary on *Vayikra* XXIII; 3 that this early visibility of the moon can happen in Nissan or in the three former months. However, he considered incorrectly this early vision of the moon one day before the *keviyah*, to be a commonplace and he wrote that it happens rather frequently.

<sup>&</sup>lt;sup>141</sup> As we still see in the letter of the *Resh Galuta*.

<sup>&</sup>lt;sup>142</sup> A year following a leap year. The first year of a cycle of 19 years is a year L+1.

<sup>&</sup>lt;sup>143</sup> Extrapolated in the past before its inception in 358 / 359 C.E.

<sup>&</sup>lt;sup>144</sup> The second year AMI is also the first year AMII, it is a year L-1, preceding a leap year.

by the Jews of Spain and Italy.<sup>145</sup> Similarly the *tekufah* of Adda, a system of mean equinox and solstices fixed rigidly to the cycle of 19 Jewish years and having a good coincidence with the mean equinox and solstices during the 10<sup>th</sup> and 11<sup>th</sup> century, was probably introduced in Spain and it was thoroughly studied by the Spanish *meabrim*. Finally the Four Gates Table,<sup>146</sup> a Babylonian discovery, was generalized by the French Tossafist Ritsva,<sup>147</sup> of the 12<sup>th</sup> century and gave birth to the table of the 61 lines, a table giving the *keviyah* of all the 19 years of a cycle by the simple knowledge of the Molad Tishrei of its first year.

It is interesting to note that this important event of 922-924 remained unknown until the beginning of the twentieth century, until the discovery and the study of the documents of the Cairo Geniza. It is a fact that R' Sherira Gaon and R' Hai Gaon did not mention the event at all. At first glance we could think that the leaders of the Babylonian community did not want to leave a remembrance of this schism for posterity; it could have thrown a shadow on the authority of the Jewish calendar and on the doctrine of its Sinaïtic origin taught by R' Sa'adia Gaon. This, however, is not the case. We know that R' Sa'adia Gaon wrote two books: ספר הזיכרון and ספר המועדים. The first book was intended to be read publicly in order to recall the event. The second book was probably a treatise on the festivals and the Jewish calendar and it probably also mentioned the events of the famous dispute of 922- 924148 in order to prevent the possibility of a new schism in the future. It was the fear of mahaloquet that prompted him to write the first and probably the second book. R' Sa'adia's works on the calendar are lost, although they appear to have been well known in the middle ages (Rashi, R' Tam and R' Jacob ben Shimshon<sup>149</sup> refer to it). It is a mystery why these two books did not survive.

<sup>&</sup>lt;sup>145</sup> The principle of beginning the counting of the Jewish years one year before the era of *Weyad* (AMII) was already discussed by R' Sa'adia Gaon and R' Hai Gaon but it was rejected by them (see Bornstein, *Mahaloket* 1904, p. 127). It must be remembered that the counting from the year of *Weyad* corresponds to the counting of the Talmud (B. Avoda Zara 9b) according to the era of the creation.

<sup>&</sup>lt;sup>146</sup> See details in Appendix C.

<sup>&</sup>lt;sup>147</sup> R' Isaac ben Abraham, elder brother of R' Samson ben Abraham of Sens. This attribution was demonstrated by Bornstein.

<sup>&</sup>lt;sup>148</sup> Encyclopedia Judaica Vol. 14, entry Sa'adiah, p.544 bottom, affirms, without evidence or reference, that the *Sefer ha-Moadim* gave a complete account of the dispute.

<sup>&</sup>lt;sup>149</sup> For details about R' Jacob ben Shimshon, the "secretary" of Rashi after R' Shemaya, see Abraham Grossman, *Hakhmei Sarfat ha-Rishonim*, (Jerusalem: Magnes, 1996) pp. 411-426.

By contrast, it is evident that the Palestinian side was not interested to speak about this event and indeed they never did mention this dispute again. It is worth mentioning that in Tishrei 4686, the Molad was 5 - 18- 214 and a new schism should have appeared about the *keviyah* of 4686. Indeed according to the Babylonians Rosh Hashanah 4686 was on Saturday and the year had the *keviyah*. But for the Palestinians the Molad must occur 642 *hal* before, at: 5 - 17 - 652 and Rosh Hashanah should have been on Thursday, with the *keviyah* Rosh Hashanah should with exactly the same problem as four years earlier.

In fact there is no information left about a new dispute about the *keviyah* of that year. It seems that the Palestinian Gaon adopted the Babylonian Molad and proclaimed the *keviyah* as usual, as if nothing occurred. Later in the *Megilat Abiathar*,<sup>150</sup> the Palestinian Gaon did not mention anything about the incident but he still claimed the Palestinian authority on the calendar.<sup>151</sup>

The present day calendar was the calendar of the Babylonians since about 838 C.E. that emerged after the dispute of 922-924. This calendar did not change any more.

In the following two tables we show the weak point of the present calendar, i.e., that the Jewish year is shifting with regard to the Gregorian calendar, in the direction of the summer. This brings us to contemplate

<sup>&</sup>lt;sup>150</sup> See "Megilat Abiathar," Schechter *JQR* Vol XIV (1901-1902) pp. 449-474.

<sup>&</sup>lt;sup>1</sup> It is also likely that the Palestinians went on calculating the Molad according to their more ancient methods referring to Nissan. Indeed Bornstein discovered that R' Jacob ben Shimshon used methods of calculation similar to that of the Palestinians in the time of Ben Meir. Similarly the Four Gates Table in *Mahzar Vitry* (Vol. 2 end) is constructed according to Nissan. It appears clearly that the French Rabbis were under the influence of the Babylonian but also the Palestinian Gaonim. We know that the German Jewish establishment was of Palestinian origin and had ties with Palestine. We are aware of the responsum of the Palestinian Gaon Elijah ben Solomon ha-Cohen, R' Abiathar's father, to R' Meshulam ben Moses of Mainz in 1070. It was also signed by R' *Abiathar harevi'i*, then the fourth in rank in the *yeshivah*. Grossman has discovered in the Library of the JTS the following passage: זצ"ל הרביעי שבחבורה בן אדונינו אביתר הכהן זצ"ל הרביעי שבחבורה בן אדונינו אביתר המ"ח ליצירה

See Grossman, *Hakhmei Tsarfat ha-Rishonim*, Magnes 1996, p. 423. This document, dated 1088, makes sense: In 1081, while his father was still alive, R' Abiathar was appointed gaon and his son Elijah (named as his still alive grand-father) was appointed the fourth in rank in the yeshiva.

again a slight adaptation of the Jewish calendar in order to remain in agreement with the solar year. This subject is beyond the scope of the present paper. It was already thoroughly examined in two other papers.<sup>152</sup>

## IX The Present-day Jewish Calendar and the rule of intercalation.

#### Table 5: The dates of Nissan 16 and the following Tishrei 21 during the 243<sup>rd</sup> cycle: Pessah was perfectly calibrated in the *hodesh ha-aviv*

Ν	Year	Jewish Year	Nissan 16	Tishrei 21
			Gregorian	Gregorian
1	2036	5796-5797	April 13	October 12
2	2037	5797-5798	April 1	September 30
3	2038	5798-5799	April 21	October 20
4	2039	5799-5800	April 10	October 9
5	2040	5800-5801	March 30	September 28
6	2041	5801-5802	April 17	October 16
7	2042	5802-5803	April 6	October 5
8	2043	5803-5804	April 26	October 25
9	2044	5804-5805	April 13	October 12
10	2045	5805-5806	April 3	October 2
11	2046	5806-5807	April 22	October 21
12	2047	5807-5808	April 12	October 11
13	2048	5808-5809	March 30	September 28
14	2049	5809-5810	April 18	October 17
15	2050	5810-5811	April 8	October 7
16	2051	5811-5812	March 29	September 27
17	2052	5812-5813	April 15	October 14
18	2053	5813-5814	April 4	October 3
19	2054	5814-5815	April 24	October 23

<sup>152</sup> Ajdler (2011), "The Future of the Jewish Calendar." *BDD* 25.
 Ajdler (2013/1), "The Gregorian Revolution of the Jewish Calendar," *BDD* 27.

Table 6:	The dates of Nissan 16 and Tishrei 21 in the 304th cycle.
	We note a shift of a few days. Pesah is no more completely
	in the <i>hodesh ha-aviv</i>

ЪT	3.7	T ' 1			TT: 1 : 01	TT: 1 : 04
Ν	Year	Jewish	Nissan 16	Nissan 16	Tishrei 21	Tishrei 21
		Year	Julian	Gregorian	Julian	Gregorian
1	839	4599-4600	April 4	April 8	October 3	October 7
2	840	4600-4601	March 24	March 28	Sept. 22	Sept. 26
3	841	4601-4602	April 11	April 15	Oct. 10	Oct. 14
4	842	4602-4603	March 31	April 4	Sept. 29	Oct. 3
5	843	4603-4604	March 21	March 25	Sept. 19	Sept. 23
6	844	4604-4605	April 7	April 11	Oct. 6	Oct. 10
7	845	4605-4606	March 27	March 31	Sept. 25	Sept. 29
8	846	4606-4607	April 16	April 20	Oct. 15	Oct. 19
9	847	4607-4608	April 6	April 10	Oct. 5	Oct. 9
10	848	4608-4609	March 25	March 29	Sept. 23	Sept. 27
11	849	4609-4610	April 12	April 16	Oct. 10	Oct. 14
12	850	4610-4611	April 2	April 6	Oct. 1	Oct. 5
13	851	4611-4612	March 22	March 26	Sept. 20	Sept. 24
14	852	4612-4613	April 10	April 14	Oct. 9	Oct. 13
15	853	4613-4614	March 29	April 2	Sept. 27	Oct. 1
16	854	4614-4615	March 18	March 22	Sept. 16	Sept. 20
17	855	4615-4616	April 7	April 11	Oct. 6	Oct. 10
18	856	4616-4617	March 27	March 31	Sept. 25	Sept. 29
19	857	4617-4618	April 14	April 18	Oct.13	Oct. 17

### Appendices<sup>153</sup>

### Appendix A The Modern Jewish Calendar

#### I **References:** See <www.Hakirah.org/vol20AjdlerAppendices.pdf>.

#### II The fundamental formula of the Jewish calendar.

A. The number of months preceding the *molad* of the Jewish year N + 1, counted from *Beharad*, is given by

#### $F_t = INT [(235N + 1)/19]$ .<sup>154</sup>

The following table gives the practical demonstration of this formula.

### Table 7: Number of months at the beginning of the year N + 1 in a cycle of 19 years.

Ν	Ft	Ν	Ft	Ν	Ft	Ν	Ft
1	12	6	74	11	136	16	197
2	24	7	86	12	148	17	210
3	37	8	99	13	160	18	222
4	49	9	111	14	173	19	235
5	61	10	123	15	185	20	247

The numbers of columns F are indeed the number of the months preceding the beginning of the different years of the cycle of 19 years. It is based on a cycle of intercalation of the years 3 - 6 - 8 - 11 - 14 - 17 - 19.

This formula is general. It allows calculating the *molad* of any year.

B. The Molad expressed as a part of the week is:

<sup>&</sup>lt;sup>153</sup> Due to space constraints the appendices to this article were shortened. The full version can be found at <www.Hakirah.org/vol20AjdlerAppendices.pdf>.

<sup>&</sup>lt;sup>154</sup> This formula was given for the first time in *Al ha-Sheminit*, Y Loewinger, Tel Aviv 1986. The formula  $\mathbf{F_t} = \mathbf{INT}$  [(235N)/19] fits except for N = 8. Indeed for N = 8, INT [(235\*8)/19]= 98 instead of 99. This is the justification of the formula  $\mathbf{F_t} = \mathbf{INT}$  [(235N + 1)/19].

 $Mol = [31524 + F_t * 765443]_{181440} = [31.524 + F_t * 39673]_{181440}$ 

31524 is the span of time between the beginning of the week, Saturday afternoon at 6 p.m. noted 1 - 0 - 0 and the moment *Beharad* or 2 - 5 - 204; 765443 is the length of the Jewish lunation 29 - 12 - 793 in *halakim* and 39673 is the rest of the division of 765443 by 181440.

#### III Converting a Jewish date into a civil date by using the Julian day.

The classical methods for converting a Jewish date into a civil date are long and dull. The principle rests on the calculation of the *tekufah* of Samuel of September with regard to Tishrei 1 and on the fact that the *tekufah* of Tishrei always falls on September 24 in the Julian calendar. Louis A, Resnikoff<sup>156</sup> described an algorithm based on the same principle applicable to pocket calculators. Another method of computation makes use of the formula of Gauss<sup>157</sup> giving the date of Nisan 15 in the Julian calendar.<sup>158</sup>

We propose here a simple method in which we calculate the molad as a moment of the week and as a precise moment in history thanks to the Julian day. The method is conceptually very simple but it must, however, be applied with care and precision.

<sup>&</sup>lt;sup>155</sup>  $[A]_B$  is the remainder of the division of A by B.

<sup>&</sup>lt;sup>156</sup> Scripta Mathematica Vol. IX, pp. 191-196 and 274-277.

<sup>&</sup>lt;sup>157</sup> Gauss, Werke VI Bd. 1874, pp. 80-81. Berechnung des Judischen Osterfestes. Zach's Monatliche Correspondenz zur beforderung der Erd und Himmelskunde, Mai 1802, p. 435.

Different authors tried to demonstrate this formula:

<sup>• &</sup>quot;Ableitung der gausschen formel zur bestimmung des Judischen Osterfestes, M. Hamburger," *Crelles Journal fur die reine und angewandte Mathematik*, Band 116 (1896).

Computation of the dates of the Hebrew New Year and Passover, Ida Rhodes, Comp. & Maths with Appls. Vol 3, pp. 183-190, Pergamon Press 1977.

<sup>•</sup> A short and elegant demonstration has been proposed by the author of this paper in J. Ajdler (2013/1).

<sup>&</sup>lt;sup>158</sup> Other formulas were proposed, for example:

Eine algemeine Formel fur die gesamte judischen Kalenderberechnung, Slonimsky aus Bialystock, Crelles Journal fur die reine und angewandte Mathematik, Band 26 (1844).

<sup>• &</sup>quot;Beitrage zur Chronologie, Nesselman in Königsberg," Crelles Journal fur die reine und angewandte Mathematik, Band 28 (1844).

Let us consider a concrete example: Nisan 15, 5751.

- 1. The characteristics of the Jewish year A = N + 1 = 5751.
  - a. The rank of the year 5751 in the cycle of 19 years.

 $[5751]_{19} = 13$ ; the year 5751 is the 13<sup>th</sup> year of the cycle 303 of 19 years; it is a regular year preceding a leap year.

b. The Molad of the year 5751.

The number of Jewish months preceding the Molad of year 5751 is given by the fundamental formula of the Jewish calendar:<sup>159</sup>

 $F_t = INT [(235N+1)/19] = INT [(235 * 5750+1)/19] = 71118.$ 

The Molad expressed as a part of the week is:

 $Mol = [31524 + 71118 * 765443]_{181440} {}^{160} = [31.524 + 71118 * 39673]_{181440} = 103938 hal = 4 - 0 - 258 = (5) - 0 - 258.$ 

This Molad is thus after 4 days and 258 *halakim* or at the beginning of the fifth day at 0h 258 *halakim* i.e. Wednesday at 18h 258 hal. Tishrei 1 falls on Thursday.

The Four Gates Table gives then the *keviyah* of the year, הכז. Rosh Hashanah is Thursday and Pesah is on Saturday.

This result can also be reached directly by calculating the Molad of the years 5751 and 5752 and the days of Tishrei 1 of these two years by the application of the four rules of postponement.

 $F_t = INT [(235 * 5751 + 1)/19] = 71130.$ 

Mol =  $[31524 + 71130 * 765443]_{181440} = 35694$  hal = 1 - 9 - 54 = (2) - 9 - 54. Tishrei 1 falls on Monday. The shift of Tishrei 1 between 5751 and 5752 is thus four days and the number of days lying between these two days, exclusive of the two days of Tishrei 1, is  $3.^{161}$  Therefore the year

<sup>&</sup>lt;sup>159</sup> See: Mathematical appendix in "The Gregorian Revolution of the Jewish Calendar", J. Ajdler (2013/1), pp. 17 - 76. See also J. Loewinger (1986).

<sup>&</sup>lt;sup>160</sup>  $[A]_B$  is the remainder of the division of A by B.

<sup>&</sup>lt;sup>61</sup> This is the algorithm described by Maimonides in *Hilkhot Kiddush ha-Hodesh VIII*, 7 and 8. He counts the number of days between the two days of Tishrei 1, exclusive of the two days of Tishrei 1. The length of the year is thus 353, 354 or 355 days according whether this difference is 2, 3 or 4 for a common year, 383, 384 and 385 according whether this difference is 4, 5 or 6 for a leap year. By contrast R. Abraham bar Hiyya counts the shift of Rosh Hashanah between the two years, i.e. he counts the day of Rosh Hashanah of one year + the number

5751 is a regular year and its length is 354 days. Thus Rosh Hashanah falls on Thursday because of the rules of the *dehiyot* (postponements) and the length of the year is 354 days.

c. The year 5751 is thus an ordinary<sup>162</sup> year; it is a regular<sup>163</sup> year of 354 days beginning on a Thursday.

Nisan 15 is the 192<sup>nd</sup> day of this year and it falls on a Saturday.<sup>164</sup>

2. The Jewish calendar and the Julian day.

The Julian period's epoch is Monday, January 1, -4712 at noon. At this moment the number of elapsed day of the Julian period was 0 days. The Julian day n° 1 began on Monday at noon and ended on Tuesday at noon. Similarly, until the twentieth century, the astronomical days began at noon of the civil days of the same name.

The Molad of *Beharad*, beginning in the Jewish era AMI, was on Sunday October 6, - 3760 at 23h 204*hal*; Jerusalem mean time. This moment already belonged to the second Jewish day of the week, which began at 18h, hence (2) – 5 – 204. It means the second day at 5 h and 204 *halakim*. It could be written as 1 - 2 - 204, meaning 1 day 5 h and 204 *hal* after the beginning of the week or 31524 *hal* after the beginning of the week.<sup>165</sup>

Expressed in Julian days, the molad of Beharad was 347997. 466203703703. On Sunday, October 6, - 3760 at noon, 347 997 days of the JP<sup>166</sup> had elapsed and on Monday, October 7, - 3760 = Tishrei 1, 1 AMI, at noon, 347 998 days of the JP had elapsed. Tishrei 1, 1 AMI began thus at 347997.25 JD and ended at 347998.25 JD. Tishrei 1 corresponded in its majority to the day 347998 of the JP.<sup>167</sup>

of days between. Therefore the length of the year is 353, 354 or 355 days according whether the difference is 3, 4 or 5 for a common year and 383, 384 or 385 according whether the shift is 5, 6 or 7 for a leap year.

<sup>&</sup>lt;sup>162</sup> An ordinary year has 12 months and a leap year has 13 months.

<sup>&</sup>lt;sup>163</sup> A regular year has 354 or 384 days, a defective year has 353 or 384 days and a full year has 355 or 385 days according to whether the year is a regular or a leap year.

<sup>&</sup>lt;sup>164</sup> See the fourteen possible calendars of the Jewish calendar: Yesodei ha-Ibbur, Hayim Zelig Slonimski, Warsaw 1852, end of the book. Shearim le-luah ha-Ivri, Rahamim Sar Shalom, Natania 5744, p. 35.

Ha-Luah ve-Shimusho ba-Kronologia, A. A. Akabia, (Jerusalem: Magnes, 1953), pp. 50-53 and E. Mahler, Handbuch Der Jüdischen Chronologie, 1915 and 1967 Hildesheim, pp. 614 – 627.

<sup>&</sup>lt;sup>165</sup> See note 114.

<sup>&</sup>lt;sup>166</sup> Julian Day.

<sup>&</sup>lt;sup>167</sup> Julian Period.

There is a second style of the Jewish calendar AMII, beginning on Tishrei 1, 2 AMI.

The molad of this year was Weyad: 6-14.

The first day of this year was Tishrei 1, 1 AMII = Tishrei 1, 2 AMI; it corresponds to Saturday, September 27, - 3759 or 348353 JD, beginning at 348352.25 JD and ending at 348353.25 JD.

We note also that Elul 25, 1 AMI = Monday, September 22, - 3759 = 348348 JD.

3. The year 5751 and the civil year.

Expressed in Julian days, the molad of 5751 is given by the formula:<sup>168</sup> Mol = 347997.466203703 + 29.530594135804 \* 71118 = 2448154.25995370370 JD

This molad is thus on a civil Wednesday 18h 258 hal or on a Jewish Thursday at 0 h 258 hal.

Rosh Hashanah is thus Thursday, from 2448154.25 JD until 2448155.25 JD.

Tishrei 1, 5751 corresponds thus to 2448155 JD and Nisan 15 = 2448155 + 191 = 2448346 JD. This day corresponds to Saturday, March 30, 1991.<sup>169</sup>

<sup>&</sup>lt;sup>168</sup> This formula gives the same result as the formula of Shram.

<sup>&</sup>lt;sup>169</sup> For the conversion of a Julian day into a civil date see Astronomical Algorithms, Jean Meeus, Willman-Bell, 1991, p. 59. Idem for the determination of the weekday.

#### Appendix B The Derivate Postponements

#### I The Derivate Postponements in the Modern Calendar

1. The postponement 3 - 9 - 204 or ג ט רד בפשוטה.

If the Molad of Tishrei of an ordinary year is 3-9-204 or greater, then the Molad of the following Tishrei is 7-18 or greater. If we apply the general rules we will begin Tishrei of the present year on Tuesday and Tishrei of next year on Monday. The shift of Rosh Hashanah from one year to the other will be 6 days and therefore the ordinary year must be a multiple of 7 plus 6, thus necessarily 356 days. This is impossible; the Jewish ordinary year must have 353, 354 or 355 days. In order to solve this difficulty we must impose to postpone the first day of Rosh Hashanah to Thursday as soon as the molad is 3-9-204 in an ordinary year.

2. The Postponement 2 – 15 – 589 or גרעיבור אחר עיבור.

If the Molad of Tishrei following a leap year 2 - 15 - 589 or more the Molad Tishrei of the preceding year is 3 - 18 or more. If we apply the general rules the 1 Tishrei of the leap year is Thursday and the 1 Tishrei of the following year is Monday. The shift from one year to the other is 4 days. The number of days of the leap year must be a multiple of 7 plus 4. It is necessarily 382 days. This is impossible; the number of days of a leap year is 383, 384 or 385 days. In order to solve this difficulty we must postpone the first day of Rosh Hashanah of a year following a leap year from Monday to Tuesday as soon as the Molad reaches 2 - 15 - 589 and this will bring the number of days of the leap year to 383 days.

#### II The Calendar of Hillel, from about 648 C.E. till 776 C.E.

The reasoning is the same. The limit 3 - 9 - 204 in an ordinary year becomes 3 - 9 - 3 or ג ט ג בפשוטה.

Similarly the limit 2 - 15 - 589 after a leap year becomes 2 - 15 - 8 after a leap year or ב טו ה אחר עיבור.

#### III The Calendar of Hillel from 359 until about 648.

1 Tishrei could be on Sunday. By similar reasoning it is easy to demonstrate that the two derivate postponements are:

1-9-3 in an ordinary year or א ט ג בפשוטה.

2 - 15 - 8 after a leap year or ב טו ה אחר עיבור.

#### Appendix C The Four Gates Table

The Four Gates Table is a Babylonian invention from the 9th century. It represents a higher degree of sophistication and knowledge of the rules of the calendar. It allows knowing the *keriyah* of a year by the knowledge of its Molad and its rank in the cycle of 19 years.

Maimonides did not describe this method in *Hilkhot Kiddush ha-Hodesh*. He must find the day of 1 Tishrei of two consecutive years in order to find the characteristics of the first year. R' Abraham ibn Ezra worked the same way in his *Sefer ha-Ibbur*.

The Four Gates Table is mentioned in a letter of R' Sa'adia Gaon related to the dispute.<sup>170</sup> He also gave the detailed rules of the Four Gates Table. We also have a description of the Four Gates Table in a poem of R' Yose ben al-Naharwani.<sup>171</sup> The Four Gates was thus well-established knowledge in Babylonia. The Four Gates Table was thoroughly examined by R' Abraham bar Hiyya in *Sefer ha-Ibbur*<sup>172</sup> and in R' Isaac Israeli's *Yessod Olam*. In the supplement at the end of the second volume of *Mahzor Vitry*<sup>173</sup> we find the table of the Four Gates according to the molad of the preceding Nissan.

<sup>&</sup>lt;sup>170</sup> See Bornstein, "Divrei Yemei ha-Ibbur ha-Aharonim," *ha-Tekufah* 16, p. 247. He accuses Ben Meir of copying the Babylonian Four Gates Table and adapting the different limits by the addition of 642 *hal.* 

<sup>&</sup>lt;sup>171</sup> Epstein, A. (1901) : La querelle au sujet du Calendrier entre Ben Meir et les académies Babyloniennes, REJ 42, pp. 173-210.

<sup>&</sup>lt;sup>172</sup> Pp. 63-69.

<sup>&</sup>lt;sup>173</sup> This supplement begins after page 798. It is likely that this chapter was greatly influenced, if not copied from the *Sefer ha Ibbur* by R' Jacob ben Samson, which was part of his great composition: the *Sefer Elkoshi*. In *Mahzor Vitry* we find also the commentary on *Avot* by R' Jacob ben Samson. Abraham Berliner, on pp. 15-16 of the calendar supplement to *Mahzor Vitry* seems to ignore that the book of R' Jacob ben Samson has the general name of *Sefer Elkoshi* and he assumes that the author of the manuscript was called Nahum according to Nahum I; 1. In any case, it seems that R' Jacob ben Samson exerted an important influence on different parts of the *Mahzor Vitry*.

# I. The Four Gates table for the modern calendar according to the Molad of Tishrei. <sup>174</sup>

Table 10: The Four Gates Table for the modern calendar. For the explanation of the precise meaning of this table, let us consider the left column devoted to the years L - 1.

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If 7 - 18 - 0 \le Molad \le 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203 the year is 2\pi - 2\pi = 1 - 9 - 203.
```

If  $1 - 9 - 204 \le Molad \le 2 - 17 - 1079$  the year is , etc.

The Four Gates Table — לוה ארבע שערים							
	Leap Years						
L-1		L+1		L+-1		L	
רבי עיבור	עו	צאי עיבור	מו:	ני עיבור	בי	נות עיבור	W
2-5-10-13	8 – 16	1 - 4 - 9 - 12	- 15	7-18		3-6-8-11	-1417
						- 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7-18-01-	2d	7 - 18 - 0	2d	7 - 18 - 0	2d	7 - 18 - 0	2D
9-203	בחג	1 - 9 - 203	בחג	1 - 9 - 203	בחג	1 - 20 - 490	בחה
1 - 9 - 204	2f	1 - 9 - 204	2f	1 - 9 - 204	2f	1 - 20 - 491	2F
2-17-1079	השב	2 - 15 - 588	בשה	2 - 15 - 588	בשה	2-17-1079	בשז
2 - 18 - 0	3r	2 - 15 - 589	3r	2 - 15 - 589	3r	2 - 18 - 0	3R
3 - 9 - 203	הכג	3 - 9 - 203	גכה	3 - 9 - 203	גכה	3 - 17 - 1079	גכז
3 - 9 - 204	5r	3 - 9 - 204	5r	3 - 9 - 204	5r	3 - 18 - 0	5D
5 - 9 - 203	הכז	5 - 9 - 203	הכז	5 - 9 - 203	הכז	4 - 11 - 694	החא
5 - 9 - 204	5f	5 - 9 - 204	5f	5 - 9 - 204	5f	4-11-695	5F
5 - 17 - 1079	השא	5-17-1079	השא	5 -17 -	השא	5 - 17 - 1079	השג
				1079			
5 - 18 - 0	7d	5 - 18 - 0	7d	5 - 18 - 0	7d	5 - 18 - 0	7D
6-9-203	זחא	6 - 0 - 407	זחא	6-9-203	זכא	6 - 20 - 490	זחג
6 - 9 - 204	7f	6 - 0 - 408	7f	6-9-204	7f	6-20-491	7F
7 - 17 - 1079	זשג	7-17-1079	זישג	7 -17 - 1079	זשג	7-17-1079	זשה

<sup>&</sup>lt;sup>174</sup> See <www.Hakirah.org/vol20AjdlerAppendices.pdf> for more details.

#### Appendix D

Calculations of Moladot of the Jewish Calendar in the period 359 C.E. – 921 C.E. considered in the present paper

### 1. The year 4119 AMI, at the inception of the calculated Jewish calendar.

#### Calculation of the modern Molad of Nissan 4119.

The fundamental formula of the modern calendar allows calculating the number of lunations elapsed from *Beharad* until the molad of the year 4119. 4119 is the 15<sup>th</sup> year of the fictitious cycle of 19 years; the preceding year was probably a leap year.

Ft = Int [(235 \* 4118 + 1) / 19] = 50933.

The number of lunations before the Molad of Nissan 4119 is then 50939. The molad of Nissan 4119 is thus:

 $Mol = [31524 + 50939 * 39673]_{181440} = 55751 hal = 2d + 3h + 671 hal = 3 - 3 - 671 thus 3h 671 hal later than the epoch adopted by Hillel: <math>3 - 0 - 0$ .

In order to make later calculations easier, we will calculate the modern Molad for the year 4124 representing the first year of the fictitious 218<sup>th</sup> cycle of intercalation (of 19 years).

The number of lunations between Beharad and Tishrei 4124 is:

Ft = Int [(235 \* 4123 + 1) / 19] = 50995.

 $Mol = [31524 + 50995 * 39673]_{181440} = 100159 = 3d+20 h+799 hal = 4 - 20 - 799.$  (Modern Molad).

The Molad of Hillel is 4 - 17 - 1 (*hayil*) = 4 - 17 - 72 *hal* 

The difference is 3h 727 hal = 3h 671hal + 50995 - 50939 = 3h 727 hal.between our modern molad and the assumed molad of Hillel. The Molad of Hillel is thus 4 - 17 - 1 (havil) = 4 - 17 - 72 hal

#### 2. Keviyah of the year 4147 AMI (386/387 C.E.).

#### Calculation of the modern Molad.

The number of lunations preceding Tishrei 4147 is:

Ft = Int[(235\*4146 + 1) / 19] = 51279.

The Molad in the modern calendar is:

$$\label{eq:Mol} \begin{split} \text{Mol} &= [31524 + 51279 * 39673]_{181440} = \! 118011 \! = 4 \text{d} \! + \! 13 \text{h} \! + \! 291 \textit{hal} \! = 5 \\ &- 13 - 291 \end{split}$$

#### Calculation of the Molad of Hillel.

In the calendar of Hillel the Molad was thus:

```
5 – 13 – 291
– 3 – 727 difference in 4124
```

$$- 284 = (51279 - 50995)$$
  
5 - 9 - 360

It corresponds perfectly to the Molad of Jaffe: 5 - 9 - 5 in his table  $\aleph$ .

The *keviyah* of the year 4147 was thus in the calendar of Hillel as it is also the case in our modern calendar: השא.

#### Molad Nissan 4147.

The year 4147 is assumed to be an ordinary year. The number of lunations preceding Nissan is thus 51279 + 6 = 51285.

The molad in the modern calendar is:

 $Mol = [31524 + 51285 * 39673]_{181440} = 174609 = 6d + 17h + 729 hal = 7$ -17 - 729

In the calendar of Hillel the Molad was thus:

Molad in the calendar of Hillel 
$$7 - 17 - 729$$
  
 $- 3 - 727$   
 $- 290 = (51285 - 50995)$   
 $7 - 13 - 792$ 

Now if we write the modern Molad in terms of the Julian Period, we get: Mol = 347997.466203703 + 29.530594135804 \* 51285 = 1862473.98645JD. Thus our modern Molad falls slightly before the beginning of the day 1862474. It corresponds to Saturday 6 March 387. But Nissan 1 was a Sunday; the Molad Nissan 387 was thus on Saturday 6 March 387, 1 Nissan was Sunday 7 March and 15 Nissan, the first day of Passover was on Sunday 21 March 387. The rule of the equinox was satisfied and therefore our assumption that it was an ordinary year is validated.

#### 3. The year 4267AMI.

Year 4267 began on Sunday. This year was the eleventh year of the fictitious cycle 224 of 19 years. It is likely that it was a leap year.

In our modern calendar the Molad of

4267 is	1 - 22 - 983
We can deduce the Molad of Hillel:	-3-727
	1768 = (52763–50995)
Molad of Hillel of year 4267:	1 - 17 - 648
It corresponds to the Molad of Jaffe	1 - 17 - 9.

If we adopt the Molad of the modern calendar, we have a Molad Zaken and 1 Tishrei could not be on Sunday but it should have been delayed to Monday. By contrast, with the Molad of Hillel, 1 - 17 - 9, there was no Molad Zaken and 1 Tishrei was indeed on Sunday.

#### 4. The year 4537 AMI (776 /777 C.E.).

The year 4537 is the 15<sup>th</sup> year of a fictitious cycle of 19 years; it is assumed to be an ordinary year. The number of lunations preceding Tishrei 4537 is given by the formula:

Thus the Molad of Hillel of Tishrei 4537 was 3 - 18 - 1008. It corresponds exactly to the Molad of Jaffe 3 - 18 -14. It was corrected after the observation of September 776 to 4 - 0 - 0 by the addition of 5 - 72, thus 5 hours and 1/15. The modern value of the corresponding Molad is 4 - 3 - 363. Thus in 776 the difference after introduction of the new epoch 4 - 0 - 0, there still was a difference of 3 - 363 with regard to the modern Molad.

#### 5. The year 4596 AMI (835 / 836 C.E.).

First assumption: The Jewish lunation is 29 - 12 - 793. The Molad Nissan 4596 is deduced from the modern Molad by subtracting 3 - 363. Thus 3 - 15 - 811 - (3 - 363) = 3 - 12 - 448.

Second assumption: The Jewish lunation is 29 -12 – 793.2962 (*Iggul de Rav Nahshon*).

The difference between the modern Molad and the ancient Molad is reduced by 0,2946 \* (56890 - 56103) = 232 hal. The Molad Nissan 4596 would then be 3 - 12 - 680 very near to the value calculated by Jaffe in his table. Similarly the Molad Tishrei 4596 was 6 - 19 - 297 or  $6 - 19 - 529.^{175}$ 

<sup>&</sup>lt;sup>175</sup> There was a Molad *Zaken* in Shevat, see Appendix H at <www.Hakirah .org/vol20AjdlerAppendices.pdf>.

#### Appendix E See <www.Hakirah.org/vol20AjdlerAppendices.pdf>.

#### Appendix F

#### Historical evidence of the existence of the keviyah kuyah ku

#### 1. R' Abraham bar Hiyya.

In his *Sefer ha-Ibbur*,<sup>176</sup> he mentions twice the *keviyah* **XWX**. He first mentions the *keviyah* as a possible *keviyah*<sup>177</sup> but later he writes that this possible theoretical *keviyah* did not find a practical application because this was not necessary.<sup>178</sup>

#### 2. Massekhet Sofrim.<sup>179</sup>

In *Massekhet Soferim* XX, 12 it deals with the reading of the Torah on both days of *Rosh Hodesh* Tevet when *Rosh Hodesh* falls on Sunday and Monday.<sup>180</sup>

There are two days of *Rosh Hodesh* if the year is regular or full. In the first assumption the first day of *Rosh Hodesh* is Tishrei 89. But if the year is full then the first day of *Rosh Hodesh* is Tishrei 90. The first assumption implies that 1 Tishrei was four weekdays before the first day of *Rosh Hodesh*. Thus if the first day of *Rosh Hodesh* is Sunday, 1 Tishrei is on Wednesday. This is impossible. The only possibility is then that we are in a full year  $\Im \lambda$ . If it is an ordinary year it has 355 = M7 + 5 days and Rosh Hashanah of next year is on Friday. This is impossible. It must then be a leap year of 385 = M7 days and next year will also begin on Tuesday.

<sup>&</sup>lt;sup>176</sup> Ed. Filipowski, London 1851.

<sup>&</sup>lt;sup>177</sup> P. 63.

<sup>&</sup>lt;sup>178</sup> P. 65.

<sup>&</sup>lt;sup>179</sup> The reference to *Massekhet Sofrim* was mentioned for the first time by Hayyim Jehiel Bornstein in "Divrei Yemei ha-Ibbur ha Aharonim," *Ha-Tekufah* 16, Warsaw, 1923, p. 283.

<sup>&</sup>lt;sup>180</sup> In the text of Massekhet Sofrim published in the Vina Romm edition and in the Massekhet Soferim edited in Mahzor Vitry, ed. Simon Horowitz, Nuremberg 1923, Vol. 2, p. 716 there is an additional interpolation, שאין השבון ראש הדש מיום שני The signification of this interpolation is that Rosh Hodesh Tevet has two days only if the year is regular (Marheshvan defective and Kislev full) or full (Marheshvan and Kislev full). This interpolation is not necessary at all and Gra suppressed it.

Pesah of this year will be two days before, on Sunday and the *keviyah* is then **NWA**. This *keviyah* does not exist today but we can assume that it once existed or, at least, that it was once taken into consideration.

3. Sefer ha-Pardes.<sup>181</sup>

*Sefer ha-Pardes* is one of the books issued by the "school of Rashi"; Berliner assumed that it was composed by R' Shemaya.

In *Sefer ha Pardes*, about the Shabbat and festivals readings,<sup>182</sup> it writes that if Sukkot is on Tuesday and Marheshvan and Kislev are full there will be 29 Sabbaths and we won't be obliged to read two sections together. The year considered is a full leap year beginning on Tuesday. It has 385 days and the next year also begins on Tuesday. Pesah will be two days before the day of Rosh Hashanah of next year, on Sunday. It is a year **NWX**. Apparently, these two quotations are remnants of ancient calendar rules which were not adapted or corrected and which fortunately could reach us.<sup>183</sup> They attest to the depth of their knowledge of the Jewish calendar.

<sup>&</sup>lt;sup>181</sup> The reference to Sefer ha-Pardes was mentioned for the first time by Hayyim Bornstein in "Divrei Yemei ha-Ibbur ha Aharonim," Ha-Tekufah 16, Warsaw, 1923, p. 273.

<sup>&</sup>lt;sup>182</sup> Sefer ha-Pardes, ed. R' H.L. Ehrenreich, Budapest 1924 and Bnei Berak 1990, p. 340 five lines from bottom.

<sup>&</sup>lt;sup>183</sup> We note that the Gra corrected the reading in Soferim XX, 12 but he did not react and note the impossibility of this configuration. It is thus normal, because of the difficulty of the subject, that the copyists copied without amending the text and let survive these interesting passages.

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