Appendices

Appendix A
The Modern Jewish Calendar

I References

Hebrew:

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II The fundamental formula of the Jewish calendar.

A. The number of month preceding the *molad* of the Jewish year N + 1, counted from *Beharad*, is given by

$$\mathbf{F_t = INT\ [(235N + 1)/19]}.$$ [1]

The following table gives the practical demonstration of this formula.

Table 1: Number of months at the beginning of the year N + 1 in a cycle of 19 years.

N	**F_t**	**N**	**F_t**	**N**	**F_t**	**N**	**F_t**
1	12	6	74	11	136	16	197
2	24	7	86	12	148	17	210
3	37	8	99	13	160	18	222
4	49	9	111	14	173	19	235
5	61	10	123	15	185	20	247

The numbers of columns F are indeed the number of the months preceding the beginning of the different years of the cycle of 19 years. It is based on a cycle of intercalation of the years 3 – 6 – 8 – 11 – 14 – 17 – 19.

This formula is general. It allows calculating the *molad* of any year.

B. The Molad expressed as a part of the week is:

$$\mathrm{Mol} = [31524 + F_t * 765443]_{181440}{}^{2} = [31.524 + F_t * 39673]_{181440}$$

31524 is the span of time between the beginning of the week, Saturday afternoon at 6 p.m. noted 1 – 0 – 0 and the moment *Beharad* or 2 – 5 – 204; 765443 is the length of the Jewish lunation 29 – 12 – 793 in *ḥalakim* and 39673 is the rest of the division of 765443 by 181440.

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1 This formula was given for the first time in *Al ha-Sheminit*, Y Loewinger, Tel Aviv 1986. The formula **F_t = INT [(235N)/19]** fits except for N = 8. Indeed for N = 8, INT [(235*8)/19]= 98 instead of 99. This is the justification of the formula **F_t = INT [(235N + 1)/19].**

2 $[A]_B$ is the remainder of the division of A by B.

III Converting a Jewish date into a civil date by using the Julian day.

The classical methods for converting a Jewish date into a civil date are long and dull. The principle rests on the calculation of the *tekufah* of Samuel of September with regard to Tishrei 1 and on the fact that the *tekufah* of Tishrei always falls on September 24 in the Julian calendar. Louis A, Resnikoff[3] described an algorithm based on the same principle applicable to pocket calculators. Another method of computation makes use of the formula of Gauss[4] giving the date of Nisan 15 in the Julian calendar.[5]

We propose here a simple method in which we calculate the molad as a moment of the week and as a precise moment in history thanks to the Julian day. The method is conceptually very simple but it must, however, be applied with care and precision.

Let us consider a concrete example: Nisan 15, 5751.

1. The characteristics of the Jewish year A = N + 1 = 5751.

   a. The rank of the year 5751 in the cycle of 19 years.

$[5751]_{19} = 13$; the year 5751 is the 13th year of the cycle 303 of 19 years; it is a regular year preceding a leap year.

   b. The Molad of the year 5751.

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3 S*cripta Mathematica* Vol. IX, pp. 191-196 and 274-277.

4 Gauss, Werke VI Bd. 1874, pp. 80-81. *Berechnung des Judischen Osterfestes.* Zach's *Monatliche Correspondenz zur beforderung der Erd und Himmelskunde*, Mai 1802, p. 435.
Different authors tried to demonstrate this formula:
- "Ableitung der gausschen formel zur bestimmung des Judischen Osterfestes, M. Hamburger," *Crelles Journal fur die reine und angewandte Mathematik*, Band 116 (1896).
- *Computation of the dates of the Hebrew New Year and Passover*, Ida Rhodes, Comp. & Maths with Appls. Vol 3, pp. 183-190, Pergamon Press 1977.
- A short and elegant demonstration has been proposed by the author of this paper in J. Ajdler (2013/1).

5 Other formulas were proposed, for example:
- *Eine algemeine Formel fur die gesamte judischen Kalenderberechnung, Slonimsky aus Bialystock, Crelles Journal fur die reine und angewandte Mathematik*, Band 26 (1844).
- "Beitrage zur Chronologie, Nesselman in Königsberg," *Crelles Journal fur die reine und angewandte Mathematik*, Band 28 (1844).

The number of Jewish months preceding the Molad of year 5751 is given by the fundamental formula of the Jewish calendar:[6]

F_t = INT [(235N+1)/19] = INT [(235 * 5750+1)/19] = 71118.

The Molad expressed as a part of the week is:

Mol = $[31524 + 71118 * 765443]_{181440}$ [7]= $[31.524 + 71118 * 39673]_{181440}$ = 103938 hal.= 4 – 0 – 258 = (5) – 0 – 258.

This Molad is thus after 4 days and 258 *ḥalakim* or at the beginning of the fifth day at 0h 258 *halakim* i.e. Wednesday at 18h 258 ḥal. Tishrei 1 falls on Thursday.

The Four Gates Table gives then the *keviyah* of the year, הכז. Rosh Hashanah is Thursday and Pesaḥ is on Saturday.

This result can also be reached directly by calculating the Molad of the years 5751 and 5752 and the days of Tishrei 1 of these two years by the application of the four rules of postponement.

F_t = INT [(235 * 5751 + 1)/19] = 71130.

Mol = $[31524 + 71130 * 765443]_{181440}$ = 35694 ḥal = 1 – 9 – 54 = (2) – 9 – 54. Tishrei 1 falls on Monday. The shift of Tishrei 1 between 5751 and 5752 is thus four days and the number of days lying between these two days, exclusive of the two days of Tishrei 1, is 3.[8] Therefore the year 5751 is a regular year and its length is 354 days. Thus Rosh Hashanah falls on Thursday because of the rules of the *dehiyot* (postponements) and the length of the year is 354 days.

c. The year 5751 is thus an ordinary[9] year; it is a regular[10] year of 354 days beginning on a Thursday.

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6 See: Mathematical appendix in "The Gregorian Revolution of the Jewish Calendar", J. Ajdler (2013/1), pp. 17 - 76. See also J. Loewinger (1986).

7 $[A]_B$ is the remainder of the division of A by B.

8 This is the algorithm described by Maimonides in *Hilkhot Kiddush ha-Ḥodesh VIII*, 7 and 8. He counts the number of days between the two days of Tishrei 1, exclusive of the two days of Tishrei 1. The length of the year is thus 353, 354 or 355 days according whether this difference is 2, 3 or 4 for a common year, 383, 384 and 385 according whether this difference is 4, 5 or 6 for a leap year. By contrast R. Abraham bar Ḥiyya counts the shift of Rosh Hashanah between the two years, i.e. he counts the day of Rosh Hashanah of one year + the number of days between. Therefore the length of the year is 353, 354 or 355 days according whether the difference is 3, 4 or 5 for a common year and 383, 384 or 385 according whether the shift is 5, 6 or 7 for a leap year.

9 An ordinary year has 12 month and a leap year has 13 months.

Nisan 15 is the 192nd day of this year and it falls on a Saturday.[11]

2. The Jewish calendar and the Julian day.

The Julian period's epoch is Monday, January 1, – 4712 at noon. At this moment the number of elapsed day of the Julian period was 0 days. The Julian day n° 1 began on Monday at noon and ended on Tuesday at noon. Similarly, until the twentieth century, the astronomical days began at noon of the civil days of the same name.

The Molad of *Beharad*, beginning in the Jewish era AMI, was on Sunday October 6, - 3760 at 23h 204*ḥal*; Jerusalem mean time. This moment already belonged to the second Jewish day of the week, which began at 18h, hence (2) – 5 – 204. It means the second day at 5 h and 204 *ḥalakim*. It could be written as 1 – 2 – 204, meaning 1 day 5 h and 204 *hal* after the beginning of the week or 31524 *hal* after the beginning of the week.[12]

Expressed in Julian days, the molad of Beharad was 347997. 466203703703. On Sunday, October 6, - 3760 at noon, 347 997 days of the JP[13] had elapsed and on Monday, October 7, - 3760 = Tishrei 1, 1 AMI, at noon, 347 998 days of the JP had elapsed. Tishrei 1, 1 AMI began thus at 347997.25 JD and ended at 347998.25 JD. Tishrei 1 corresponded in its majority to the day 347998 of the JP.[14]

There is a second style of the Jewish calendar AMII, beginning on Tishrei 1, 2 AMI.

The molad of this year was *Weyad*: 6- 14.

The first day of this year was Tishrei 1, 1 AMII = Tishrei 1, 2 AMI; it corresponds to Saturday, September 27, - 3759 or 348353 JD, beginning at 348352.25 JD and ending at 348353.25 JD.

We note also that Elul 25, 1 AMI = Monday, September 22, - 3759 = 348348 JD.

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10 A regular year has 354 or 384 days, a defective year has 353 or 384 days and a full year has 355 or 385 days according to whether the year is a regular or a leap year.

11 See the fourteen possible calendars of the Jewish calendar: *Yesodei ha-Ibbur*, Hayim Zelig Slonimski, Warsaw 1852, end of the book. *Shearim le-luah ha-Ivri*, Rahamim Sar Shalom, Natania 5744, p. 35.
Ha-Luah ve-Shimusho ba-Kronologia, A. A. Akabia, (Jerusalem: Magnes, 1953), pp. 50-53 and E. Mahler, *Handbuch Der Jüdischen Chronologie*, 1915 and 1967 Hildesheim, pp. 614 – 627.

12 See note **Error! Bookmark not defined.**.

13 Julian Day.

14 Julian Period.

3. The year 5751 and the civil year.

Expressed in Julian days, the molad of 5751 is given by the formula:[15]

Mol = 347997.466203703 + 29.530594135804 * 71118 = 2448154.25995370370 JD

This molad is thus on a civil Wednesday 18h 258 hal or on a Jewish Thursday at 0 h 258 ḥal.

Rosh Hashanah is thus Thursday, from 2448154.25 JD until 2448155.25 JD.

Tishrei 1, 5751 corresponds thus to 2448155 JD and Nisan 15 = 2448155 + 191 = 2448346 JD. This day corresponds to Saturday, March 30, 1991.[16]

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15 This formula gives the same result as the formula of Shram.

16 For the conversion of a Julian day into a civil date see Astronomical Algorithms, Jean Meeus, Willman-Bell, 1991, p. 59. Idem for the determination of the weekday.

Appendix B
The Derivate Postponements

I The Derivate Postponements in the Modern Calendar

1. The postponement 3 – 9 – 204 or **ג ט רד בפשוטה** .

If the Molad of Tishrei of an ordinary year is 3 – 9 – 204 or greater, then the Molad of the following Tishrei is 7 – 18 or greater. If we apply the general rules we will begin Tishrei of the present year on Tuesday and Tishrei of next year on Monday. The shift of Rosh Hashanah from one year to the other will be 6 days and therefore the ordinary year must be a multiple of 7 plus 6, thus necessarily 356 days. This is impossible; the Jewish ordinary year must have 353, 354 or 355 days. In order to solve this difficulty we must impose to postpone the first day of Rosh Hashanah to Thursday as soon as the molad is 3 – 9 – 204 in an ordinary year.

2. The Postponement 2 – 15 – 589 or **ב טו תקפט אחר עיבור.**

If the Molad of Tishrei following a leap year 2 – 15 – 589 or more the Molad Tishrei of the preceding year is 3 – 18 or more. If we apply the general rules the 1 Tishrei of the leap year is Thursday and the 1 Tishrei of the following year is Monday. The shift from one year to the other is 4 days. The number of days of the leap year must be a multiple of 7 plus 4. It is necessarily 382 days. This is impossible; the number of days of a leap year is 383, 384 or 385 days. In order to solve this difficulty we must postpone the first day of Rosh Hashanah of a year following a leap year from Monday to Tuesday as soon as the Molad reaches 2 – 15 – 589 and this will bring the number of days of the leap year to 383 days.

II The Calendar of Hillel, from about 648 C.E. till 776 C.E.

The reasoning is the same. The limit 3 – 9 – 204 in an ordinary year becomes 3 – 9 – 3 or ג ט ג בפשוטה.

Similarly the limit 2 – 15 – 589 after a leap year becomes 2 – 15 – 8 after a leap year or ב טו ח אחר עיבור.

III The Calendar of Hillel from 359 until about 648.

1 Tishrei could be on Sunday. By similar reasoning it is easy to demonstrate that the two derivate postponements are:

1 – 9 – 3 in an ordinary year or א ט ג בפשוטה.

2 – 15 – 8 after a leap year or ב טו ח אחר עיבור.

Appendix C
The Four Gates Table

The Four Gates Table is a Babylonian invention from the 9th century. It represents a higher degree of sophistication and knowledge of the rules of the calendar. It allows knowing the *keviyah* of a year by the knowledge of its Molad and its rank in the cycle of 19 years.

Maimonides did not describe this method in *Hilkhot Kiddush ha-Hodesh*. He must find the day of 1 Tishrei of two consecutive years in order to find the characteristics of the first year. R' Abraham ibn Ezra worked the same way in his *Sefer ha-Ibbur*.

The Four Gates Table is mentioned in a letter of R' Sa'adia Gaon related to the dispute.[17] He also gave the detailed rules of the Four Gates Table. We also have a description of the four gates table in a poem of R' Yose ben al-Naharwani.[18] The Four Gates was thus well-established knowledge in Babylonia. The Four Gates Table was thoroughly examined by R' Abraham bar Ḥiya in *Sefer ha-Ibbur*[19] and in R' Isaac Israeli's *Yessod Olam*. In the supplement at the end of the second volume of *Maḥzor Vitry*[20] we find the table of the Four Gates according to the molad of the preceding Nissan.

I. Construction of the Four Gates Table for the Modern Calendar.

We depart from the daily limits of each type of year.

17 See Bornstein, "Divrei Yemei ha-Ibbur ha-Aharonim," *ha-Tekufah* 16, p. 247. He accuses Ben Meir of copying the Babylonian four gates table and adapting the different limits by the addition of 642 *ḥal*.

18 Epstein, A. (1901) : La querelle au sujet du Calendrier entre Ben Meir et les académies Babyloniennes, REJ 42, pp. 173-210.

19 Pp. 63-69.

20 This supplement begins after page 798. It is likely that this chapter was greatly influenced, if not copied from the *Sefer ha Ibbur* by R' Jacob ben Samson, which was part of his great composition: the *Sefer Elkoshi*. In *Maḥzor Vitry* we find also the commentary on *Avot* by R' Jacob ben Samson. Abraham Berliner, on pp. 15-16 of the calendar supplement to *Mahzor Vitry* seems to ignore that the book of R' Jacob ben Samson has the general name of *Sefer Elkoshi* and he assumes that the author of the manuscript was called Nahum according to Nahum I; 1. Anyhow, it seems that R' Jacob ben Samson exerted an important influence on different parts of the *e*.

Table 2: The Modern Calendar. Table of the different limits in Tishrei according to the weekdays. The limits are noon of the permissible days for Rosh Hashanah. However, the limit of 18 hours is replaced by *Gatrad* in ordinary years and by *Betoutakpat* in years following a leap year.

Type	Monday	Tuesday	Thursday	Saturday
L	7 – 18	2 – 18	3 – 18	5 – 18
L+1	7 – 18	2 – 15 – 589	3 – 9 – 204	5 – 18
L-1	7 – 18	2 – 18	3 – 9 – 204	5 – 18
L±1	7 – 18	2 – 15 – 589	3 – 9 – 204	5 – 18

L represents the Leap years, i.e. the years 3, 6, 8, 11, 14, 17, 19.
L+1 represents the years following a leap year i.e. 1 – 4 – 9 – 12 – 15.
L–1 represents the years preceding a leap year i.e. 2 – 5 -10 – 13 – 16.
L±1 represents the years, which follow and in the same time precede a leap year i.e. 7 – 18.
The different limits of the table are 18hours of the permissible days for Rosh Hashanah. But in ordinary years 3 – 18 is replaced by 3 – 9 – 204 and in a year following a leap year 2 – 18 is replaced by 2 – 15 – 589.

We already have 4 limits for each category of year. In order to find the three additional limits we proceed as follows:[21]

- We subtract from the first line the remainder of 12 months 4 – 8 – 876. We get the complementary limits of the third line L – 1:[22]
  1 – 9 – 204, 5 – 9 – 204 and 6 – 9 – 204.
- We subtract from the second line the remainder of 13 months 5 – 21 – 589 and we get the additional limits of the line L:[23]
  1 – 20 – 491, 4 – 11 – 695 and 6 – 20 – 491

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21 See <https://en.wikibooks.org/wiki/Computer_Programming/Hebrew_Calendar> for an alternative presentation of the Jewish calendar and of the Four Gates table.

22 For example in a year L – 1, if the molad is 1 – 9 – 204, then the molad of the following year is
(1 – 9 – 204) + (4 – 8 – 876) = 5 – 18. Therefore Tishri 1 of the following year is delayed from Thursday to Saturday and the considered year L – 1, which was בחג becomes בשה.

23 For example in a year L, if the molad is 1 – 20 – 491, then the molad of the following year is
(1 – 20 – 491) + (5 – 21 – 589) = 7 – 18. Therefore Tishri 1 of the following year is delayed from Saturday to Monday and the considered year L, which was החב becomes בשז.

- We subtract from the third line the remainder of 12 months 4 – 8 – 876 and we get the additional limits of the line L+1:[24]
  1 – 9 – 204, 5 – 9 – 204 and 6 – 0 – 408.
- We subtract from the limits of the first line the remainder of the length of 12 months 4 – 8 – 876. We get the complementary limits of the third line L±1:[25] 1– 9 – 204, 5 – 9 – 204 and 6 – 9 – 204.

The Four Gates Table for the modern calendar according to the Molad of Tishrei.

Table 10: The Four Gates Table for the modern calendar.

The Four Gates Table — לוח ארבע שערים							
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 20 – 490	2D בחה
1 – 9 – 204 2 – 17 – 1079	2f השב	1 – 9 – 204 2 –15 – 588	2f בשה	1 – 9 – 204 2 –15 – 588	2f בשה	1 – 20 – 491 2 –17- 1079	2F בשז
2 – 18 – 0 3 – 9 – 203	3r הכג	2 – 15 – 589 3 – 9 – 203	3r גכה	2 – 15 – 589 3 – 9 – 203	3r גכה	2 – 18 – 0 3 – 17–1079	3R גכז
3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 18 – 0 4 – 11 – 694	5D החא
5 – 9 – 204 5 – 17 – 1079	5f השא	5 – 9 – 204 5 –17– 1079	5f השא	5 – 9 – 204 5 –17 –1079	5f השא	4 – 11 – 695 5 – 17–1079	5F השג
5 – 18 – 0 6 – 9 – 203	7d זחא	5 – 18 – 0 6 – 0 – 407	7d זחא	5 – 18 – 0 6 – 9 – 203	7d זכא	5 – 18 – 0 6 – 20 – 490	7D זחג
6 – 9 – 204 7 – 17 – 1079	7f זשג	6 – 0 – 408 7 –17– 1079	7f זשג	6 – 9 – 204 7 –17– 1079	7f זשג	6 – 20 – 491 7 –17– 1079	7F זשה

24 The year following a year L+1 is generally a year L – 1 except when the year L+1 is also a year L – 1.
For example, in a year L +1, if the molad is 1 – 9 – 204, then the molad of the following year is
(1 – 9 – 204) + (4 – 8 – 876) = 5 – 18. Therefore Tishri 1 of the following year is delayed from Thursday to Saturday and the considered year L + 1, which was בחג becomes בשה.

25 For example in a year L ± 1, if the molad is 1 – 9 – 204, then the molad of the following year is
(1 – 9 – 204) + (4 – 8 – 876) = 5 – 18. Therefore Tishri 1 of the following year is delayed from Thursday to Saturday and the considered year L ± 1, which was בחג becomes בשה.

For the explanation of the precise meaning of this table, let us consider the left column devoted to the years L – 1. If 7 – 18 – 0 <= Molad <=1 – 9 – 203 the year is בחג. If 1 – 9 – 204 <= Molad <=2 – 17 – 1079 the year is בשה. And so on

II. The Four gates table for the presumed calendar of Hillel: 648-776.[26]

Table 13: The Four Gates Table for the presumed calendar of Hillel: 648-776.

The Four Gates Table — לוח ארבע שערים							
Ordinary Years						**Leap Years**	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 2	2d בחג	7 – 18 – 0 1 – 9 – 2	2d בחג	7 – 18 – 0 1 – 9 – 2	2d בחג	7 – 18 – 0 1 – 20 – 6	2D בחה
1 – 9 – 3 2 – 17 – 14	2f השב	1 – 9 – 3 2 –15 – 7	2f השב	1 – 9 – 3 2 –15 – 7	2f השב	1 – 20 – 7 2 –17 – 14	2F בשז
2 – 18 – 0 3 – 9 – 2	3r הכג	2 – 15 – 8 3 – 9 – 2	3r הכג	2 – 15 – 8 3 – 9 – 2	3r הכג	2 – 18 – 0 3 – 17 –14	3R גכז
3 – 9 – 3 5 – 9 – 2	5r הכז	3 – 9 – 3 5 – 9 – 2	5r הכז	3 – 9 – 3 5 – 9 – 2	5r הכז	3 – 18 – 0 4 – 11 – 9	5D החא
5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 –17– 14	5f השא	5 – 9 – 3 5 –17 –14	5f השא	4 – 11 –10 5 – 17 –14	5F השג
5 – 18 – 0 6 – 9 – 2	7d זחא	5 – 18 – 0 6 – 0 – 5	7d זחא	5 – 18 – 0 6 – 9 – 2	7d זכא	5 – 18 – 0 6 – 20 – 6	7D זחג
6 – 9 – 3 7 – 17 – 14	7f זשג	6 – 0 – 6 7 –17– 14	7f זשג	6 – 9 – 3 7 –17– 14	7f זשג	6 – 20 – 7 7 –17– 14	7F זשה

III. The Four Gates table for the presumed calendar of Hillel: 358-648.

We depart from the daily limits of each type of year. We already have 5 limits for each category of year. In order to find the four additional limits we proceed as follows:

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26 The Construction of the Four Gates Table for the Calendar of the Period 648-776 can be easily deduced from the four gates table for the modern calendar by replacing 3 – 9 – 204 by 3 – 9 – 3 and 2 – 15 – 589 by 2 – 15 – 8.

Table 3: The Calendar of Hillel Table of the different natural limits in Tishrei according to the days of the week.

Type	Sunday	Monday	Tuesday	Thursday	Saturday
L	7 – 18	1 – 18	2 – 18	3 – 18	5 – 18
L+1	7 – 18	1 – 9 – 3	2 – 15 – 8	3 – 18	5 – 18
L-1	7 – 18	1 – 9 – 3	2 – 18	3 – 18	5 – 18
L ±1	7 – 18	1 – 9 – 3	2 – 15 – 8	3 – 18	5 – 18

The 4 Types of years (column 1) refer to specific years in the 19 year cycle as follows:

L represents the Leap years, i.e. the years 3, 6, 8, 11, 14, 17, 19.
L+1 represents the years following a leap year i.e. 1 – 4 – 9 – 12 – 15.
L–1 represents the years preceding a leap year i.e. 2 – 5 – 10 – 13 – 16.
L±1 represents the years which, follow and in the same time precede, a leap year i.e. 7 – 18.

The different limits of the table are 18 hours of the permissible days for Rosh Hashanah. But in ordinary years 1 – 18 is replaced by 1 – 9 – 3 and in a year following a leap year 2 – 18 is replaced by 2 – 15 – 8.

- We subtract from the limits of the first line the remainder of 12 months 4 – 8 – 12. We get the complementary limits of the third line L – 1: 3 – 9 – 3, 4 – 9 – 3, 5 – 9 – 3and 6 – 9 – 3.[27]
- We subtract from the limits of the 2nd line the remainder of 13 months 5 – 21 – 8 and we get the additional limits of the line L:
  1 – 20 – 7, 2 – 11 – 10, 4 – 20 – 7 and 6 – 20 – 7[28]
- We subtract from the limits of the third line the remainder of 12 months 4 – 8 – 12. We get the additional limits of the line L+1:[29]
  3 – 9 – 3, 4 – 0 – 6, 5 – 9 – 3 and 6 – 9 – 3.

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27 For example, in a year L – 1, if the molad is 3 – 9 – 3, then the molad of the following year is (3 – 9 – 3) + (4 – 8 – 12) = 7 – 18. Thus Tishri 1 of the following year is delayed from Saturday to Sunday and the considered year L – 1, which was גכה becomes גשו.

28 For example in a year L, if the molad is 1 – 20 – 7, then the molad of the following year is (1 – 20 – 7) + (5 – 21- 8) = 7 – 18. Thus Tishri 1 of the following year is delayed from Saturday to Sunday and the considered year L, which wasבחה becomes בכו.

29 The year following a year L+1 is generally a year L – 1 except when the year L+1 is also a year L – 1. For example in a year L+1, if the molad is 3 – 9 – 3 the molad of the following year is (3 – 9 – 3) + (4 – 8 – 12) = 7 – 18. Thus Tishri 1 of the following year is delayed from Saturday to Sunday and the considered year L+1, which was גכה becomes גשו.

- We subtract from the limits of the first line the remainder of 12 months 4 – 8 – 12. We get the complementary limits of the third line L±1: 3 – 9 – 3, 4 – 9 – 3, 5 – 9 – 3 and 6 – 9 – 3.[30]

Table 12: The Four Gates table for the presumed calendar of Hillel, 358-648.

The Four Gates Table — לוח ארבע שערים							
Ordinary Years						**Leap Years**	
L – 1		**L + 1**		**L+ – 1**		**L**	
ערבי עיבור		**מוצאי עיבור**		**ביני עיבור**		**שנות עיבור**	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 2	1r גבא	7 – 18 – 0 1 – 9 – 2	1r גכא	7 – 18 – 0 1 – 9 – 2	1r גכא	7 – 18 – 0 1 – 17 – 14	1R הכא
1 – 9 – 3 2 – 17 – 14	2f השב	1 – 9 – 3 2 – 17 - 14	2f השב	1 – 9 – 3 2 – 17 – 14	2f השב	1 – 18 – 0 1 – 20 – 6	2D בחה
2 – 18 – 0 3 – 9 – 2	3r הכג	2 – 15 – 8 3 – 9 – 2	3r הכג	2 – 15 – 8 3 – 9 – 2	3r הכג	1 – 20 – 7 2 – 11 – 9	2R בכו
3 – 9 – 3 3 – 17 – 14	3f ושג	3 – 9 – 3 3 – 17 – 14	3f ושג	3 – 9 – 3 3 – 17 – 14	3f ושג	2 – 11 – 10 2 – 17 – 14	2F בשז
3 – 18 – 0 4 – 9 – 2	5d החו	3 – 18 – 0 4 – 0 – 5	5d החו	3 – 18 – 0 4 – 9 – 2	5d וכה	3 – 18 – 0 4 – 9 – 2	5d וכה
4 – 9 – 3 5 – 9 – 2	5r זכה	4 – 9 – 3 5 – 9 – 2	5d וכה	4 – 9 – 3 5 – 9 – 2	5d החו	2 – 18 – 0 3 – 17 – 14	3R גכז
5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 – 17 – 14	5F השג
5 – 18 – 0 6 – 9 – 2	7d אחז	5 – 18 – 0 6 – 9 – 2	7d אחז	5 – 18 – 0 6 – 9 – 2	7d אחז	5 – 18 – 0 6 – 20 – 6	7D זחג
6 – 9 – 3 7 – 17 – 14	7f גשז	6 – 9 – 3 7 – 17 – 14	7f גשז	6 – 9 – 3 7 – 17 – 14	7f גשז	6 – 20 – 7 7 – 17 – 14	7F זשה

30 For example in a year L±1, if the molad is 3 – 9 – 3 the molad of the next year is (3 – 9 – 3) + (4 – 8 – 12) = 7 – 18. Thus Tishri 1 of the following year is delayed from Saturday to Sunday and the considered year L±1, which was גכה becomes גשז.

Appendix D
Calculations of Moladot of the Jewish Calendar in the period 359 C.E. – 921 C.E. considered in the present paper

1. **The year 4119 AMI, at the inception of the calculated Jewish calendar.**

Calculation of the modern Molad of Nissan 4119.

The fundamental formula of the modern calendar allows calculating the number of lunations elapsed from *Beharad* until the molad of the year 4119. 4119 is the 15th year of the fictitious cycle of 19 years; the preceding year was probably a leap year.
Ft = Int [(235 * 4118 + 1) / 19] = 50933.
The number of lunations before the Molad of Nissan 4119 is then 50939. The molad of Nissan 4119 is thus:
Mol = $[31524 + 50939 * 39673]_{181440}$ = 55751 *ḥal* = 2d + 3h + 671 *ḥal* =3 – 3 – 671 thus 3h 671 *ḥal* later than the epoch adopted by Hiliel: 3 – 0 – 0.
In order to make later calculations easier, we will calculate the modern Molad for the year 4124 representing the first year of the fictitious 218th cycle of intercalation (of 19 years).
The number of lunations between *Beharad* and Tishrei 4124 is:
Ft = Int [(235 * 4123 + 1) / 19] = 50995.
Mol = $[31524 + 50995 * 39673]_{181440}$ = 100159 = 3d+20 h+799 hal = 4 – 20 – 799. (Modern Molad).
The Molad of Hillel is 4 – 17 – 1 (*ḥayil*) = 4 – 17 – 72 *ḥal*
The difference is 3h 727 *ḥal* = 3h 671*ḥal* + 50995 – 50939 = 3h 727 *ḥal*. between our modern molad and the assumed molad of Hillel.
The Molad of Hillel is thus 4 – 17 – 1 (*ḥayil*) = 4 – 17 – 72 *ḥal*

2. ***Keviyah* of the year 4147 AMI (386/387 C.E.).**

Calculation of the modern Molad.
The number of lunations preceding Tishrei 4147 is:
Ft = Int[(235*4146 + 1) / 19] = 51279.
The Molad in the modern calendar is:
Mol = $[31524 + 51279 * 39673]_{181440}$ =118011= 4d+13h+291*ḥal* = 5 – 13 – 291
Calculation of the Molad of Hillel.
In the calendar of Hillel the Molad was thus:
5 – 13 – 291

– 3 – 727 difference in 4124
– 284 = (51279 – 50995)
----
5 – 9 – 360

It corresponds perfectly to the Molad of Jaffe: 5 – 9 – 5 in his table א.

The *keviyah* of the year 4147 was thus in the calendar of Hillel as it is also the case in our modern calendar: השא.

Molad Nissan 4147.

The year 4147 is assumed to be an ordinary year. The number of lunations preceding Nissan is thus 51279 + 6 = 51285.

The molad in the modern calendar is:

Mol = $[31524 + 51285 * 39673]_{181440}$ =174609=6d+17h+729 *hal* = 7 – 17 – 729

In the calendar of Hillel the Molad was thus:

	7 – 17 – 729
	– 3 – 727
	– 290 = (51285 – 50995)

Molad in the calendar of Hillel	7 – 13 – 792

Now if we write the modern Molad in terms of the Julian Period, we get: Mol = 347997.466203703 + 29.530594135804 * 51285 = 1862473.98645 JD. Thus our modern Molad falls slightly before the beginning of the day 1862474. It corresponds to Saturday 6 March 387. But Nissan 1 was a Sunday; the Molad Nissan 387 was thus on Saturday 6 March 387, 1 Nissan was Sunday 7 March and 15 Nissan, the first day of Passover was on Sunday 21 March 387. The rule of the equinox was satisfied and therefore our assumption that it was an ordinary year is validated.

3. The year 4267AMI.

Year 4267 began on Sunday. This year was the eleventh year of the fictitious cycle 224 of 19 years. It is likely that it was a leap year.

In our modern calendar the Molad of 4267 is	1 – 22 – 983
We can deduce the Molad of Hillel:	– 3 – 727
	1768 = (52763–50995)

Molad of Hillel of year 4267:	1 – 17 – 648
It corresponds to the Molad of Jaffe	1 – 17 – 9.

If we adopt the Molad of the modern calendar, we have a *Molad Zaken* and 1 Tishrei could not be on Sunday but it should have been delayed to

Monday. By contrast, with the Molad of Hillel, 1 – 17 – 9, there was no *Molad Zaken* and 1 Tishrei was indeed on Sunday.

4. The year 4537 AMI (776 /777 C.E.).

The year 4537 is the 15th year of a fictitious cycle of 19 years; it is assumed to be an ordinary year. The number of lunations preceding Tishrei 4537 is given by the formula:

Ft = Int [(235 * 4536+1) / 19] = 56103.

The modern Molad is given by:

Mol = $[31524 + 56103 * 39673]_{181440}$ =81363=3d+3h+363*hal*

= 4 – 3 – 363

Modern Molad	4 – 3 – 363.
	– (3 – 727)
In the calendar of Hillel	
the Molad was thus	– 5108 = (56103 – 50995)

	3– 18 –1008= 3 – 18 - 14

Thus the Molad of Hillel of Tishrei 4537 was 3 – 18 – 1008. It corresponds exactly to the Molad of Jaffe 3 – 18 -14. It was corrected after the observation of September 776 to 4 – 0 – 0 by the addition of 5 – 72, thus 5 hours and 1/15. The modern value of the corresponding Molad is 4 – 3 – 363. Thus in 776 the difference after introduction of the new epoch 4 – 0 – 0, there still was a difference of 3 – 363 with regard to the modern Molad.

5. The year 4596 AMI (835 / 836 C.E.).

First assumption: The Jewish lunation is 29 – 12 – 793. The Molad Nissan 4596 is deduced from the modern Molad by subtracting 3 – 363.
Thus 3 – 15 – 811 – (3 – 363) = 3 – 12 – 448.

Second assumption: The Jewish lunation is 29 -12 – 793.2962 (*Iggul de Rav Naḥshon*).

The difference between the modern Molad and the ancient Molad is reduced by 0,2946 * (56890 – 56103) = 232 hal. The Molad Nissan 4596 would then be 3 – 12 – 680 very near to the value calculated by Jaffe in his table. Similarly the Molad Tishrei 4596 was 6 – 19 – 297 or 6 – 19 – 529.[31]

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31 There was a Molad *Zaken* in Shevat, see Appendix H.

Appendix E
The Problem of the Molad Zaken in Shevat in a leap year.

1. Molad *Zaken* in Shevat.

The remainder of the four first months, Tishrei, Marḥeshvan, Kislev and Tevet is 6 – 2 – 1012. If the Molad of Tishrei is 1 – 15 – 67 the molad of the following Shevat is 7 – 17 – 1079. As soon as the Molad Tishrei reaches 1 – 15 – 68 the Molad Shevat reaches 7 – 18 and the Molad of Shevat is *Zaken*. If we examine the Four Gates Table we acknowledge that this can happen only in a leap year. Thus, in a leap year, if the Molad of the year is greater than 3 – 15 – 67 the Molad Shevat becomes *Zaken*. This is also the case, in a leap year, when the Molad Tishrei becomes greater than 6 – 15 – 67. The Molad of Shevat is *Zaken* because it occurs after noon of the first day of Shevat. On the other weekdays, the limits 2 – 15 – 68, 4 – 15 – 68, 5 – 15 – 68 and 7 – 15 – 68 do not have the same consequences because the leap year then becomes full and therefore there is no Molad *Zaken* because it occurs on the last day of Tevet. The Gour Gates Tables shows us also that when the Molad reaches the limit 1 – 20 – 491, 3 – 18 and 6 – 20 – 491 the leap year which was until now defective or regular, becomes full and therefore there is no more *Molad Zaken*.

We have thus a *Molad Zaken* in Shevat of a leap year if the Molad is between 1 – 15 – 68 and 1 – 20 – 490, the two boundaries included. Similarly we have a *Molad Zaken* in Shevat in a leap year if the Molad is between the 3 – 15 – 68 and 3 – 17 – 1079 and if it is between 6 – 15 – 68 and 6 – 20 – 490, the two boundaries included. Indeed, if 1 – 15 – 67 < M < 1 – 20 – 491, the leap year is defective and has the *keviyah* **בחה**. The length of four Jewish lunations is 118d 2h and 1012hal. The length of the four first months is 117 = 16 * 7 + 5 days. 1 Tishrei is a Monday and 1 Shevat is Saturday. The Molad Shevat is (1 – 15 – 68) + (6 – 2 – 1012) = 7 – 18; it is thus *Zaken*.

As soon as M = 1 – 20 – 491 the year becomes full and 1 Shevat shifts backwards by one day and there is no more *Molad Zaken*.

If 3 – 15 – 68 < M < 3 – 18, the leap year is regular and has the *keviyah* of **גכז**. The length of the four Jewish lunations is 118d 2h and 1012hal. The length of the four first months is 118 = 16 * 7 + 6. 1 Tishrei is Tuesday and 1 Shevat is Monday. The Molad of Shevat is (3 – 15 – 68) + (6 – 2 – 1012) = 2 – 18, and the Molad is *Zaken*. As soon as

M = (3 – 18) the first day of Tishrei shifts from Tuesday to Thursday and there is no more *Molad Zaken* in Shevat.

If 6 – 15 – 68 < M < 6 – 20 – 491, the leap year is defective and has the *keviyah* is **חגז**

The length of the four Jewish lunations is 118d 2h and 1012*ḥal*. The length of the first four months is 117 = 16 * 7 + 5. 1 Tishrei is Saturday and 1 Shevat is Thursday. The Molad of Shevat is (6 – 15 – 68) + (6 – 2 – 1012) = 5 – 18; the Molad is *Zaken*. As soon as M = (6 – 20 – 491) the leap year which was defective becomes full and there is no more *Molad Zaken*.

We note further that if M = 1 – 15 – 68 then the Molad of next Tishrei is: (1 – 15 – 68) + (5 – 21 – 589)[32] = 7 – 12 – 657. Similarly if M = (3 – 15 – 68) then the Molad of next Tishrei is 2 – 12 – 657 and, on the same way, if M = (6 – 15 – 68) then the Molad of next Tishrei is 4 – 12 – 657.

2. Prevention of the occurrence of a *Molad Zaken*

If we want to prevent the occurrence of a *Molad Zaken* in a leap year we must in the Four Gates Table replace the limits 1 – 20 – 491 by 1 – 15 – 68, 3 – 18 by 3 – 15 – 68 and 6 – 20 – 491 by 6 – 15 – 68.

We must make other changes; the Molad of the next Tishrei corresponding to the limits 1 – 15 – 68, 3 – 15 – 68 and 6 – 15 – 68 are 7 – 12 – 657, 2 – 12 – 657 and 5 – 12 – 657.

Thus in the lines L + 1(מוצאי עיבור)and L + – (ביני עיבור) we must replace 7 – 18 by 7 – 12 – 657, 2 – 15 – 589 by 2 – 12 – 657 and 5 – 18 by 5 – 12 – 657.

The construction of the Four Gates Table creates an additional *keviyah* in the leap years: גשא.

32 The remaining of 13 months.

Appendix F
Historical evidence of the existence of the *keviyah* גשא.

1. R' Abraham bar Hiya.

In his *Sefer ha-Ibbur*,[33] he mentions twice the *keviyah* גשא. He first mentions the *keviyah* as a possible *keviyah*[34] but later he writes that this possible theoretical *keviyah* did not find a practical application because this was not necessary.[35]

2. *Massekhet Sofrim*.[36]

In *Massekhet Soferim* XX, 12 it deals with the reading of the Torah on both days of *Rosh Ḥodesh* Tevet when *Rosh Ḥodesh* falls on Sunday and Monday.[37]

There are two days of *Rosh Ḥodesh* if the year is regular or full. In the first assumption the first day of *Rosh Ḥodesh* is Tishrei 89. But if the year is full then the first day of *Rosh Ḥodesh* is Tishrei 90. The first assumption implies that 1 Tishrei was four weekdays before the first day of *Rosh Ḥodesh*. Thus if the first day of *Rosh Ḥodesh* is Sunday, 1 Tishrei is on Wednesday. This is impossible. The only possibility is then that we are in a full year גש. If it is an ordinary year it has 355 = M7 + 5 days and Rosh Hashanah of next year is on Friday. This is impossible. It must then be a leap year of 385 = M7 days and next year will also begin on Tuesday. Pesaḥ of this year will be two days before, on Sunday and the *keviyah* is then **גשא**.This *keviyah* does not exist today but we can assume that it once existed or, at least, that it was once taken into consideration.

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[33] Ed. Filipowski, London 1851.

[34] P. 63.

[35] P. 65.

[36] The reference to *Massekhet Sofrim* was mentioned for the first time by Ḥayyim Jeḥiel Bornstein in "Divrei Yemei ha-Ibbur ha Aharonim," *Ha-Tekufah* 16, Warsaw, 1923, p. 283.

[37] In the text of *Massekhet Sofrim* published in the Vina Romm edition and in the *Massekhet Soferim* edited in *Maḥzor Vitry*, ed. Simon Horowitz, Nuremberg 1923, Vol. 2, p. 716 there is an additional interpolation, שאין חשבון ראש חדש מיום שני אלא בזמן שהשנים כסדרן. The signification of this interpolation is that *Rosh Ḥodesh* Tevet has two days only if the year is regular (Marḥeshvan defective and Kislev full) or full (Marḥeshvan and Kislev full). This interpolation is not necessary at all and Gra suppressed it.

3. *Sefer ha-Pardes.*[38]

Sefer ha-Pardes is one of the books issued by the "school of Rashi"; Berliner assumed that it was composed by R' Shemaya.

In *Sefer ha Pardes*, about the Shabbat and festivals readings,[39] it writes that if Sukkot is on Tuesday and Marḥeshvan and Kislev are full there will be 29 Sabbaths and we won't be obliged to read two sections together. The year considered is a full leap year beginning on Tuesday. It has 385 days and the next year also begins on Tuesday. Pesaḥ will be two days before the day of Rosh Hashaah of next year, on Sunday. It is a year **גשא**. Apparently, these two quotations are remnants of ancient calendar rules which were not adapted or corrected and which fortunately could reach us.[40] They attest to the depth of their knowledge of the Jewish calendar.

4. There was a *Molad Zaken* in Shevat 4596 according to their calendar.

The year 4596 is a leap year. Its modern Molad is 6 – 22 – 660 and the current molad was about 3h 20 m before: 6 – 19 – 297. This Molad is greater than 6 – 15 – 68 and there was a *Molad Zaken* in Shevat 4596. According to the exegesis of the letter of the *Resh Galuta* made by Jaffe, the *Resh Galuta* knew the *keviyah* and therefore he must find a plausible reason to the writing of this letter. Jaffe supposed that the object of the discussion was whether we accept *Molad Zaken* in Shevat or not.

The Four Gates Table of the modern calendar in the assumption that we want to prevent *Molad Zaken* in Shevat.

[38] The reference to *Sefer ha-Pardes* was mentioned for the first time by Ḥayyim Bornstein in "Divrei Yemei ha-Ibbur ha Aharonim," *Ha-Tekufah* 16, Warsaw, 1923, p. 273.

[39] *Sefer ha-Pardes*, ed. R' H.L. Ehrenreich, Budapest 1924 and Bnei Berak 1990, p. 340 five lines from bottom.

[40] We note that the Gra corrected the reading in Soferim XX, 12 but he did not react and note the impossibility of this configuration. It is thus normal, because of the difficulty of the subject, that the copyists copied without amending the text and let survive these interesting passages.

Table 13: The Four gates table of the modern calendar in the assumption that we want to prevent *Molad Zaken* in Shevat.

The Four Gates Table — לוח ארבע שערים							
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 12 –657 1 – 9 – 203	2d בחג	7 – 12 –657 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 15 – 67	2D בחה
1 – 9 – 204 2 – 17 – 1079	2f השב	1 – 9 – 204 2 –12 – 656	2f השב	1 – 9 – 204 2 –12 – 656	2f השב	1 – 15 – 68 2 –17- 1079	2F בשז
2 – 18 – 0	3r	2 – 12 – 657	3r	2 – 12 – 657	3r	2 – 18 – 0 3 – 9 – 203	3R גכז
3 – 9 – 203	הכג	3 – 9 – 203	הכג	3 – 9 – 203	הכג	3 – 15 – 68 3 – 17–1079	3F גשא
3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 3 – 9 – 203	5r הכז	3 – 18 – 0 4 – 11 – 694	5D החא
5 – 9 – 204 5 – 17 – 1079	5f השא	5 – 9 – 204 5 – 12 – 656	5f השא	5 – 9 – 204 5 –12 – 656	5f השא	4 – 11 – 695 5 – 17–1079	5F השג
5 – 18 – 0 6 – 9 – 203	7d זחא	5 –12 – 657 6 – 0 – 407	7d זחא	5 – 12 – 657 6 – 9 – 203	7d זכא	5 – 18 – 0 6 – 15 – 67	7D זחג
6 – 9 – 204 7 – 17 – 1079	7f זשג	6 – 0 – 408 7 – 12– 656	7f זשג	6 – 9 – 204 7 –12 – 656	7f זשג	6 – 15 – 68 7 –17– 1079	7F זשה

Since we know today that the reading of Jaffe in the letter of the *Resh Galuta* was incorrect,[41] there is no longer any justification to this approach.

The *Resh Galuta* did not know the Molad of the Palestinians and the problem raised was probably the contradiction between the Molad of the Palestinians and the mean conjunction deduced from the Almagest.

41 Jaffe read: ארבע ידות instead of ארבע שעות.

Appendix G:The fictitious or proleptic Gregorian calendar extrapolated until the era of *Beharad.*

The Gregorian calendar offers a good approximation of the tropical year; it is slightly too long but the difference is only one day after 3333 years. Thus an extrapolation of the Gregorian calendar backwards until the Jewish era mundi would give us a much better apprehension of the situation of some dates with regard to the seasons. It can be demonstrated that during the period – 3800 – – 3700 the difference between the two calendars was – 30 days instead of +13 days during the period 1900 – 2100.

Thus the tekufah of Samuel, which was Tuesday 25 March – 3759 at 6 p.m. was in the fictitious Gregorian calendar 30 days earlier on Tuesday 23 February and the tekufah of Adda , which was on Tuesday 1 April – 3759 was in the fictitious Gregorian calendar on 2 March. If we consider that the Gregorian calendar accumulates a difference of nearly two days in 6000 years with regard to the tropical year we can consider that the true equinox was then, at the beginning of the Jewish era of *Beharad*, rather on March 22 and the mean equinox, very roughly, on March 24. Therefore the *tekufah* of Samuel was 29 days before the mean equinox and the *tekufah* of Adda was 22 days before.

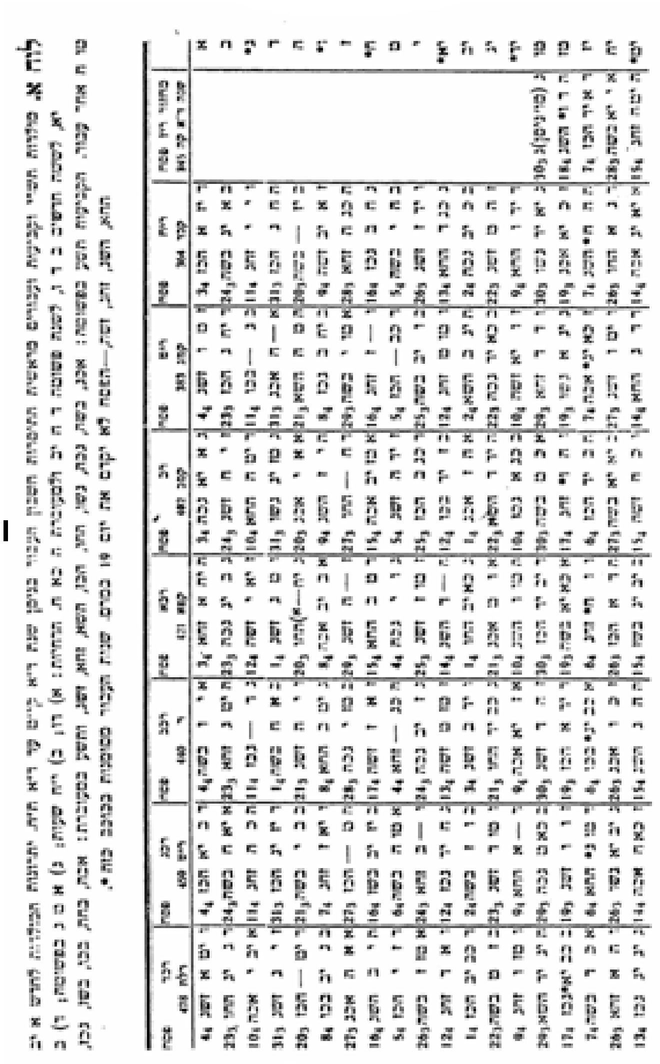
Passing from the Julian calendar to the Gregorian calendar			
– 4000 until – 3900	– 32	– 1000 until – 900	– 9
– 3900 until – 3800	– 31	– 900 until – 800	– 8
– 3800 until – 3700	– 30	– 800 until – 700	– 8
– 3700 until – 3600	– 29	– 700 until – 600	– 7
– 3600 until – 3500	– 29	– 600 until – 500	– 6
– 3500 until – 3400	– 28	– 500 until – 400	– 5
– 3400 until – 3300	– 27	– 400 until – 300	– 5
– 3300 until – 3200	– 26	– 300 until – 200	– 4
– 3200 until – 3100	– 26	– 200 until – 100	– 3
– 3100 until – 3000	– 25	– 100 until – 0	– 2
– 3000 until – 2900	– 24	0 until – 100	– 2
– 2900 until – 2800	– 23	100 until 200	– 1
– 2800 until – 2700	– 23	200 until 300	0
– 2700 until – 2600	– 22	300 until 400	1
– 2600 until – 2500	– 21	400 until 500	1
– 2500 until – 2400	– 20	500 until 600	2
– 2400 until – 2300	– 20	600 until 700	3
– 2300 until – 2200	– 19	700 until 800	4
– 2200 until – 2100	– 18	800 until 900	4
– 2100 until – 2000	– 17	900 until 1000	5
– 2000 until – 1900	– 17	1000 until 1100	6
– 1900 until – 1800	– 16	1100 until 1200	7
– 1800 until – 1700	– 15	1200 until 1300	7
– 1700 until – 1600	– 14	1300 until 1400	8
– 1600 until – 1500	– 14	1400 until 1500	9
– 1500 until – 1400	– 13	1500 until 1600	10
– 1400 until – 1300	– 12	1600 until 1700	10
– 1300 until – 1200	– 11	1700 until 1800	11
– 1200 until – 1100	– 11	1800 until 1900	12
– 1100 until – 1000	– 10	1900 until 2100	13

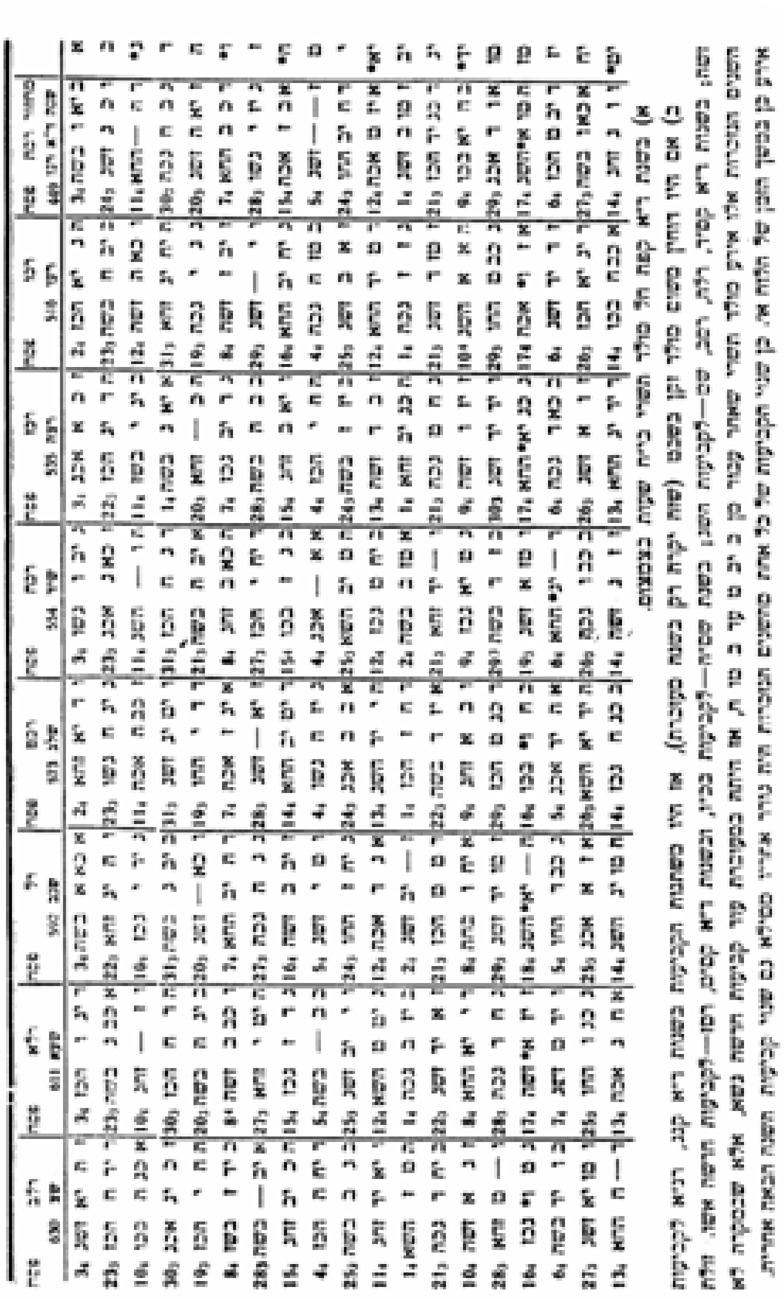
Table 14: Passing from the Julian to the Gregorian Calendar.

Appendix H: The historical tables of Moladot and Keviyot proposed by Jaffe.

I. The historical tables of Moladot and Keviyot proposed by Jaffe.

1. The period 359 – 648 C.E: **לוח א'**
2. The period 648 – 776 C.E: **לוח ב'**
3. The period 781 – 848 C.E: **לוח ה'**


לוח א.

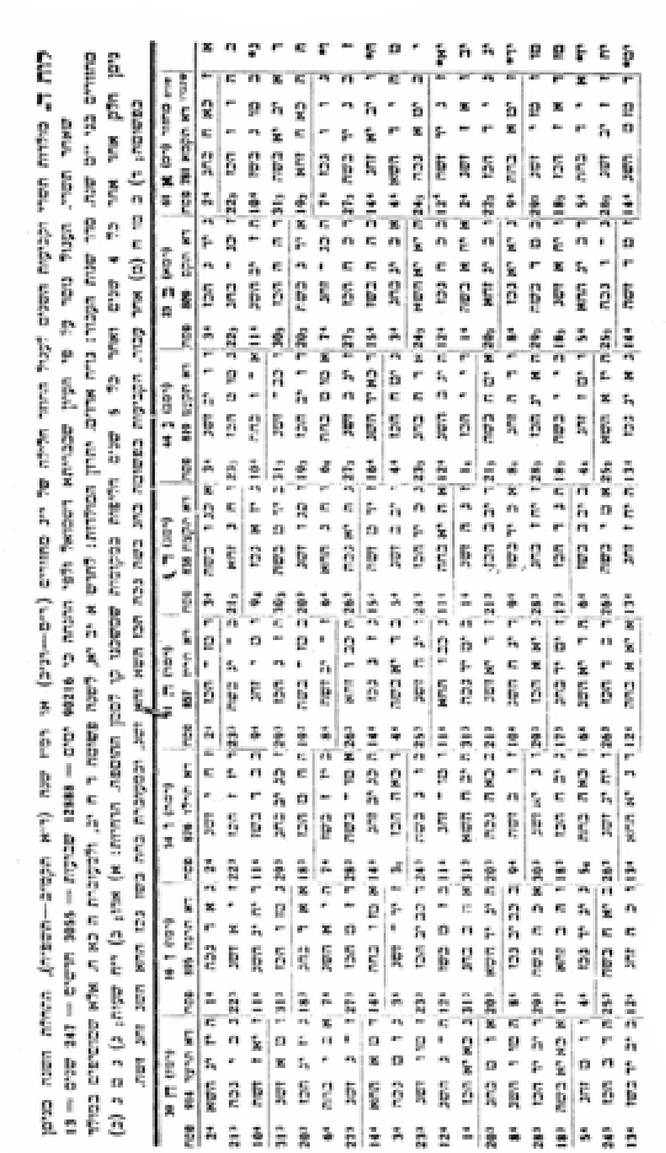


לוח ב. לשנות ר״א שצ עד ר״א תקל״ו.

יתרונות המולדות לחדש א יב יא; לשנה פשוטה ד ח יב ולמעוברת ה כא ת. הדחיות: א) אד״ו; ב) י״ח שעות; ג) ג ט ג בפשוטה; ד) ב טו ח אחר עבור. הקביעות שבע בפשוטה: בחג, בשה, נכה, הכז, חשא, זחא, ושג, ושבע במעוברת: בחה, בשז, גכז, החא, חשג, זחג, ושה. הפסח לא יקדם את י״ם 19 בסים, שנות העבור מסומנות בכוכב כזה *).

	מחזור רלב שנה ר״א שצ 630	פסח	רלג תמ 649	פסח	רלד תסח 668	פסח	רלה תפז 687	פסח	רלו תשו 706	פסח	רלז תשכה 725	פסח	רלח תשמד 744	פסח	רלט תשסג 763	פסח
א	ז ה יא ושג	3_4	ב כב א נכה	2_4	ה יד ו חשא	2_4	א ו יא בחג	2_4	ג כב א הכז	3_4	ז טו ו ושג	3_4	ב ז יא בשה	2_4	ה — א הכז	2_4
ב	ד יח ה הכז	23_5	ו ו יג ושג	23_5	ב כב ג נכה	22_5	ה טו ח חשא	22_5	א ז יג בחג	22_5	ד — ג הכז	23_5	ו טו ח ושג	23_5	ב ח יג בשה	22_5
ג*	א כב ה בשז	11_4	ד טו י השג	12_4	ז ח — ושה	11_4	ג — ה נכז	10_4	ה טו י חשג	10_4	א ט — בחה	10_4	ד א ה החא	10_4	ו יז י זחג	9_4
ד	ז כ יב בחג	30_5	ג יג ג הכז	31_5	ז ח ח ושג	1_6	א כא יב בשה	31_5	ד יד ג הכז	30_5	ו ו ח ושג	30_5	ב כב יב נכה	30_5	ה טו ג חשא	30_5
ה	ה ה י הכז	19_5	ז כב — בחג	19_5	ג יד ה הכז	20_5	ו ו י זחא	19_5	א כב — בשה	20_5	ד טו ה הכז	19_5	ו ו י ושג	19_5	ג — — נכה	19_5
ו*	ב יד ו בשז	8_4	ה ו יב השג	8_4	א כב ב בחה	7_4	ג טו ו נכז	6_4	ו ו יב זחג	7_4	ב — ב בשז	8_4	ד טו ו חשג	8_4	ו ח יב ושה	7_4
ז	א יב — בשה	28_5	ד ד ח הכז	28_5	ז כ ו ושג	28_5	ב יב — בשה	27_5	ה ה ח הכז	26_5	ז כא ו בחג	27_5	ג יד — הכז	28_5	ו ה ח זחא	28_5
ח*	ה כ יב זחג	15_4	א יג ב בחה	14_4	ד ה ז החא	15_4	ו כא יב ושה	16_4	ב יד ב בשז	15_4	ה ו ז חשג	15_4	ז כב יב בחה	15_4	ג טו ב נכז	14_4
ט	ד יח ה הכז	4_6	ו י י ושג	4_6	ג — ג נכה	3_6	ה יט ה זחא	4_6	א יא י בשה	5_6	ד ד — הכז	4_6	ו כ ה ושג	4_6	ב יב י בשה	4_6
י	ג ג ב בשה	25_5	ה יט ז הכז	24_5	א יא יב ושג	24_5	ג ד ב נכה	23_5	ה כ ז זחא	24_5	א יב יב בשה	25_5	ה ה ב הכז	24_5	ז כא ז ושג	24_5
יא*	ז יא יד זחג	11_4	ב ד ד בשז	11_4	ה כ ט חשג	13_4	ז יב יד ושה	12_4	ג ה ד נכז	11_4	ה כא ט זחג	12_4	א יג יד בחה	11_4	ד ו ד החא	11_4
יב	ה ט ז חשא	1_6	א א יב בחג	31_5	ג יח ב הכז	2_6	ו י ז ושג	2_6	ב ב יב בשה	1_6	ד יט ב הכז	31_5	ז יא ז ושג	1_6	ג ג יב נכה	31_5
יג	ב יח ד נכה	21_5	ה י ט חשא	21_5	א ב יד בחג	20_5	ג יט ד הכז	22_5	ו יא ט ושג	22_5	ב ג יד בשה	21_5	ד כ ד הכז	20_5	ז יב ט ושג	21_5
יד*	ז ג א ושה	10_4	ב יט ו נכז	9_4	ה יא יא חשג	9_4	א ד א בחה	8_4	ג כ ו החא	9_4	ז יב יא זחג	8_4	ב ה א בשז	9_4	ד כא ו חשג	9_4
טו	ד — ט ושג	30_5	א טז יד בשה	30_5	ד ח ד הכז	29_5	ז א ט בזש	29_5	ב יז יד גכה	28_5	ה י ד חשא	29_5	א ב ט בחג	28_5	קביעות השנים, ר״א תקל״ו—תקס״א עי׳ בלוח ג׳ בגוף הספר עמוד פ״ב.	
טז	ג ט ו הכז	19_5	ו א יא* זחג	16_4	א יח א בשה	19_5	ג י ו* החא	16_4	ו ב יא* ושה	17_4	ב יט א* נכז	16_4	ה יא ו* חשג	17_4		
יז	ב יח ג* בחה	6_4	ד כב ד הכז	5_4	ב יג* זחג	5_4	ג ו יד נכה	5_4	ו — ד זחא	5_4	א טו ט בשה	6_4	ד ח יד הכז	5_4		
יח	ו טו יא ושג	27_5	ב ח א בשה	26_5	ה — ו הכז	25_5	ו טו יא ושג	25_5	ג ט א נכה	25_5	ו א ו זחא	25_5	א יז יא בשה	26_5		
יט*	ד — ח החא	13_4	ו טו יב זחג	13_4	ב ט ג בשז	14_4	ה א ח חשג	14_4	ז יב יג ושה	13_4	ג ד ג נכז	13_4	ב ה ח זחג	13_4		

אם היו דוחין משום מולד זקן בשכם, או היו משתנות הקביעות בשנים מעוברות אלו: בשנת ר״א שפ״ח—אל בשו (אלו היה מחזור רל״ נכנס בלוח ב): בשנת ר״א תכג, תקכה—אל ושה, ובשנת ר״א תניב, תקל—לקביעות חרשה גשא.



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